The Radiative Transfer in Neutron star atmospheres with Super-strong Magnetic Fields

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We numerically calculate the radiative transfer in neutron star atmospheres threaded by a uniform superstrong magnetic field ($\sim 10^9 Tesla$), where not only cyclotron resonant scattering but also photon splitting occurs. Here, the multiangle radiative transfer equations are solved for cyclotron resonant scattering, which depends strongly on an angle of photon propagation with respect to the magnetic field line. The structure of the cyclotron lines are investigated in the self-emitting atmospheres with some super-strong magnetic fields around $4.41 \times 10^9 Tesla$ including photon splitting effects. We find that the cyclotron lines are sufficiently influenced by photon splitting at the cyclotron energy of 511keV in which photon splitting become effective. It is also found that the cyclotron absorption lines become shallow due to photon splitting. Furthermore, the spectra could have a common shape even for plasmas with various temperatures in the direction perpendicular to the magnetic field.

Key words: Radiative Transfer, Neutron Stars, Soft Gamma Repeaters

1.INTRODUCTION

Recently, neutron star atmospheres with extremely strong magnetic field of the order of the hundreds teragauss attract attention. First, the 1979 March 5 burst (GRB 790305) occurred within supernova (SN) remnant N49 in the Large Magellanic Cloud suggests that Soft gamma repeaters (SGRs) could have a surface dipole field of the order of $B \simeq 10^{10} T$[1]. Second, Paczynski[2] pointed out that the observed luminosities of SGRs may be sub-Eddington for sufficiently large magnetic fields, using the magnetically reduced opacity. The required fields are given by $B > \frac{\omega m_e c}{e}(\frac{2L}{L_0})^{1/2}$ where $L_0 \approx 2\times10^{38} ergs^{-1}$ is the zero-field Eddington limit. The luminosities of some SGRs (SGR 1806-20 or SGR 0525-66) require $B > 10^{10} \approx 10^{12} T$. Such strong fields permit the process of magnetic photon splitting $\gamma \rightarrow \gamma\gamma$ to act effectively below the threshold of single photon pair production $\gamma \rightarrow e^+e^-$, which is about 1 MeV. The radiative transfer problems in such strong magnetic fields were solved by Baring[3][4] SGRs are events that have episodes of short (≈ 1 s), soft (≈ 30 keV), intense (≈ 100 Crab), gamma-ray bursts (GRBs)[5]. The SGRs are distinct from classic GRBs, in that the typical photon energy is 30 keV and the events repeat. The key character is that SGR spectra have similarities between bursts from the same source.

Cyclotron lines have been detected in the spectra of accreting pulsars and GRBs. It is also possible to detect these lines in the spectra of SGRs, which are phenomena similar to GRBs. A large number of numerical calculations[6]~[13] regarding the cyclotron lines of pulsars and GRBs have been performed using the Monte Carlo method or the Feautrier method, but the spectra for neutron star atmospheres with superstrong magnetic field ($\approx 10^9 T$) taking into account cyclotron resonant scattering have not yet been calculated.

We calculate the radiative transfer in self-

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emitting atmospheres of neutron stars with superstrong magnetic field ($\approx 10^9 T$) including cyclotron resonance scattering and photon splitting. Since these are strongly dependent on the angle of photon propagation with respect to the magnetic field line, we solve the radiative transfer problem using multiangles. Although the polarization and relativistic effects are ignored for simplicity, we need to solve the radiative transfer equation taking into account the relativistic effects for plasmas with temperatures of $kT > 50keV$. We also ignored Compton scattering of the continuum component, since photon splitting effects will be dominant there. In self-emitting atmospheres with super-strong magnetic fields ($\approx 10^9 T$), we discuss how influences photon splitting give in the continuum of the spectra. We also investigate the angle-dependence on the spectrum, since photon splitting depends severly on the photon propagation angle to the magnetic field line direction. In paticular, we investigate the deformation of the cyclotron lines due to photon splitting. However, the polarization effects on cyclotron resonant scattering and photon splitting are considered to be significant. We will calculate the energy spectra including the polarization effect in the future.

2.MAGNETIC PHOTON SPLITTING AND CYCLOTRON RESONANT SCATTERING

On considering neutron star atmospheres with super-strong magnetic fields ($\approx 10^9 T$), we cannot neglect magnetic photon splitting effect, $\gamma \rightarrow \gamma\gamma$. This effect become important only in magnetic fields approaching the quantum critical value, $B_{cr} = 4.41 \times 10^9 T$. The photon splitting is phenomenon predicted by quantum electrodynamics (QED), and is forbidden in field-free regions by the charge symmetry of the theory (the Furry theorem[14]). The first calculations of the rate of photon splitting were made by Skobov[15] and Minguzzi[16]. Since these, however, were incorrect, the first correct evaluations of the reaction rate were performed by Adler et al.[17] and Bialynicka-Birula and Bialynicki-Birula[18]. These calculations were performed in the limit of zero dispersion: $\omega B/B_{cr} \leq 1$, because the reaction rate for the splitting is substantially complicated by dispersive effects caused by the deviation of the refractive index from unity in the strong field. Adler[19] demonstrated that splitting could effectively operate below the pair production threshold of $\epsilon sin\theta = 2$ in neutron star magnetospheres. Mitrofanov et al.[20] and Baring[21] discussed the possible application of $\gamma \rightarrow \gamma\gamma$ to GRB models. Baring[3] also investigated the spectral formation of γ-ray bursts from neutron stars by photon splitting effects. It was found that photon splitting produces observable effects in the continuum spectra of gamma-ray burst sources for the magnetic field $B \geq 1.5 \times 10^8 T$. Moreover, Baring[22] found that magnetic photon splitting degrades two-photon annihilation lines in neutron star sources, assuming the field-free two-photon annihilation spectrum of a simple Gaussian. Baring[4] shows that details of the reprocessing of gamma-ray radiation into the SGR energy range, by applying photon splitting to a quasi-monoenergetic injection with the energy $2m_ec^2$.

Since photon splitting effect is negligible for the photon of the cyclotron energy corresponding to the magnetic field strength ($\approx 10^8 T$), we have not ever considered photon splitting in the calculation of the cyclotron lines in such a field. The photon splitting, however, is not negligible for the photon of the cyclotron energy corresponding to a super-strong magnetic field ($\approx 10^9 T$).

3.TRANSFER EQUATION

When performing the calculations, a static, plane-parallel, isothermal atmosphere was assumed. The magnetic field direction was taken to be perpendicular to the surface of the plane-parallel atmosphere. We also ignored the polarization effects. Using the Feautrier method,[23] the radiative transfer equation can be written as

$$\mu^2 \frac{\partial^2 u(z,\mu,x)}{\partial \tau(z,\mu,x)^2} = u(z,\mu,x) - S(z,\mu,x), \quad (1)$$

where $u(z,\mu,x)$ is given by

$$u(z,\mu,x) = \frac{I(z,\mu,E) + I(z,-\mu,E)}{2} \quad (\mu > 0). \quad (2)$$

Here, $I(z,\mu,E)$ is the radiation intensity, $\kappa(\mu,E)$ is the opacity , $S(z,\mu,E)$ is the source function, Here, z is the height of the slab, E is the radiation energy, and μ is the cosine of the angle θ between the direction of the magnetic field and the line of sight to the viewer. The optical depth $[\tau(z,\mu,x)]$, that is, the mean free path of a photon for scattering and absorption is defined as $d\tau(z,\mu,x) = -\kappa(\mu,x)\rho dz$, Also, x is the deviation from the cyclotron energy in units of the Doppler width of the line $x = \frac{E-E_L}{E_L\sqrt{2kT/m_e c^2}}$. and E_L is the energy of the cyclotron lines, $E_L = \frac{\hbar e B}{m} = 11.6 \frac{B}{10^8 Tesla}\ keV$.

We took into account of cyclotron resonant scattering and photon splitting as scattering effects. The free-free absorption in strong magnetic field was considered as absorption term[24]. We used the redistribution function and the cross section in cyclotron resonant scattering derived at Wasserman and Salpeter.[25] The method for solving the cyclotron resonant scattering is similar to that of Meszaros and Nagel[7]. As for the photon splitting rate, Papanyan and Ritus[26] derived the total rate averaged over photon polarizations in the $B \ll B_c$ limit. This can be conveniently expressed[3] as an optical depth τ_{sp} for a radiation emission region of size R and further extended to consider $B \geq B_c$ regimes:

$$\tau_{sp} = 7.43 \times 10^5 (\frac{B}{B_c})^6 (\frac{\hbar\omega}{m_e c^2})^5 (1-\mu^2)^3. \quad (3)$$

In an extremely strong magnetic field ($B \approx 4.414 \times 10^9\ Tesla$), photons with energies of $\hbar\omega \geq 511 keV$ split before escaping from the emission region, although this is strongly dependent on θ. The produced photons emerge at an angle θ to the field since the splitting is a collinear process in the nondispersive limit. Figure 1 shows the optical depth of both the cyclotron scattering and photon splitting. They both depend strongly on the angle θ and the energy.

We see that photon splitting dominate cyclotron scattering at energies greater than about 511 keV in the direction normal to the field line.

The source function is, therefore, given by

$$S(z,\mu,x) = \frac{\kappa_e}{\kappa_{total}} \int\int dx' d\mu' R(x,\mu;x',\mu') \cdot u(z,\mu',x') + \frac{\kappa_a}{\kappa_{total}} B(x) + \frac{\kappa_{sp}}{\kappa_{total}} \int_x^\infty dx' \tau_{sp}(x',x) u(z,\mu,x')$$

Here, $R(x,\mu;x',\mu')$ and $\tau_{sp}(\epsilon,\omega)$ represent the redistribution function of cyclotron resonant scattering[25] and photon splitting[4], respectively. In the present calculations we employ the boundary conditions of self-emitting atmosphere. The inner boundary condition is one of perfect reflection, and the outer one assumes no incoming radiation from outside.

Assuming no radiation from above at $z{=}z_{max}$, then

$$\mu\frac{\partial u(z,\mu,x)}{\partial\tau(z,\mu,x)} = u(z_{max},\mu,x) \qquad at \quad z = z_{max}. \qquad (4)$$

If radiation is incident from neither below nor above (self-emitting slab) at z=0, then

$$\mu\frac{\partial u(z,\mu,x)}{\partial\tau(z,\mu,x)} = 0 \qquad at \quad z = 0. \qquad (5)$$

4. RESULTS AND DISCUSSION

We calculated the spectra using a set of 50 frequencies and 8 angles, with $\hbar\omega_{cyc} = 511keV$. Only thermal emission in the slab itself was considered, i.e., we used the boundary conditions (4) and (5). A density of $\rho = 1.0 \times 10^3 kg \cdot m^{-3}$, and a height of $z = 10^3 m$ was assumed . Figure 2 shows the spectra considering and non-considering the photon splitting for the direction perpendicular to **B** ($\mu \approx 0.095$) in a self-emitting atmosphere with a temperature of kT=50 keV.

The absorption lines around $\hbar\omega = 511keV$ due to cyclotron resonance scattering appear in this direction. The cyclotron absorption lines, however, is very shallow, since the cross section of cyclotron resonance is porportional to $\frac{1}{B}$ and polarization effects are ignored. Photon splitting is effective in this direction, because the cross section is proportional to $sin^6\theta$. The high energy photon is degraded down and then the structure of the cyclotron line is deformed by photon splitting. Thus,

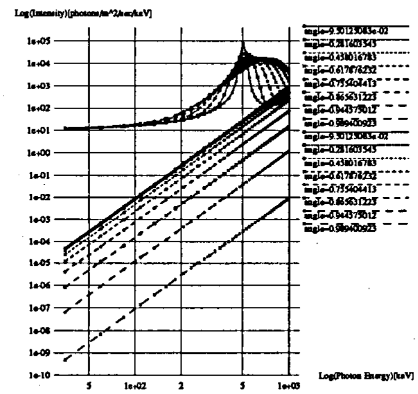

Figure 1 The opacities of cyclotron resonant scattering and photon splitting are shown for eight angles at kT=50 keV , $\rho = 1.0 \times 10^3 kg \cdot m^{-3}$ and $\hbar\omega_{cyc} = 511keV$. The eight angles are given by $\mu = cos\theta$.

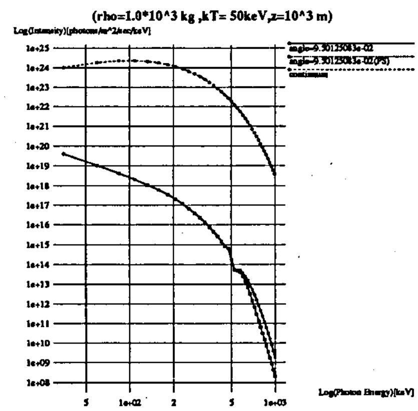

Figure 2 Spectra with and without photon splitting are shown for $\mu = cos\theta = 0.095$ in a self-emitting atmosphere at a temperature of kT=50 keV. with the magnetic field coressponging to $\hbar\omega_{cyc} = 511keV$. The depth of the slab was chosen as z=10^3 m. PS is photon splitting.

we see that the cyclotron resonance isn't effective under this condition; we might find no cyclotron line, which is consistent with the observed spectra.

Figure 3 shows the same spectra as Figure2, but for the direction inclined to the magnetic field ($\mu \approx 0.458$). We see that splitting effect is smaller than that of Fig.2. This is because the photon splitting effect is dependent on $sin^6\theta$. We see that photon splitting doesn't have considerable influence upon the structure of the cyclotron line and the continuum in this angle. The spectrum has cutoff at around $\hbar\omega = 511keV$ due to photon splitting.

Furthermore, the spectra from five different Prognoz 9 bursts in SGR 1806-20 were consistent with optically thin thermal bremsstrahlung with a single temperature, even though the total intensities of the bursts in that analysis varied by a factor of 4. We see that the spectra of plasmas at various temperatures have similar profiles. Figure 4 shows the spectra calculated for a plasma at three temperatures, with an angle of $\mu \approx 0.095$, assuming a density of $\rho = 1.0 \times 10^3 kg \cdot m^{-3}$, and a height of $z = 10^3 m$.

These spectra have a common shape in the higher energy region due to photon splitting effect. Photon splitting is independent on the temperature of the plasma. This can cause the similar spectra even in the plasma of distinct temperatures. The higher temperature is, the larger the high energy photon decreases. This is because there are a large number of photons at higher temperature which will split. This characteristic is in agreement with the common spectral shape observed by Prognoz 9.

In conclusion, we find that the photon splitting effects influences the spectrum in the self-emitting atmospheres with the superstong magnetic field. The higher component of the spectra distorted by photon splitting have similar forms for distinct temperatures of plasma, since the nature of photon splitting is to degrade the high energy photon. We will solve the radiative transfer equation including polarization, relativistic effects and

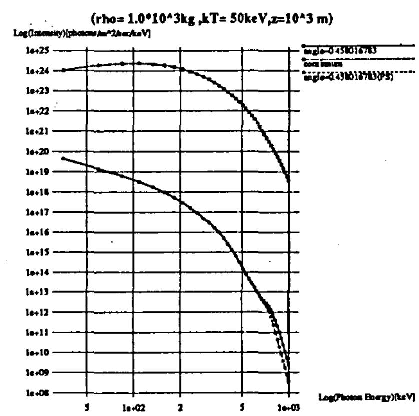

Figure 3 Same as Fig. 2, but for $\mu = cos\theta = 0.458$.

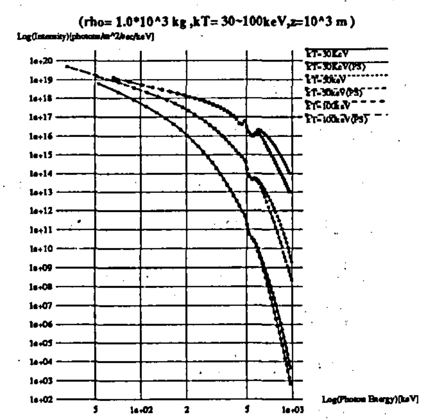

Figure 4 Spectra calculated with the photon splitting effect in a plasma at three temperatures of atmospheres for an angle of $\mu \approx 0.095$. Three temperatures assumed here are kT= 30, 50, 100 keV, respectively. PS is photon splitting.

Compton scattering of the continuum component in the feature, in order to investigate the detailed structure of the cyclotron lines and continuum spectra.

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