

The Influence of Photon Splitting on Cyclotron Line Formation in Atmospheres with Distinct Formation Regions

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The cyclotron lines are affected by photon splitting in super-strong magnetized atmospheres ($\sim 4.4 \times 10^9$ T) into which the power-law-type flux is injected. Since photon splitting occurs in super-strong magnetic field regardless of the existence of the plasma, it will occur over the larger scale than the cyclotron resonant scattering, which is interaction with the electron. We considered the atmospheres consisted of cyclotron resonant scattering predominant and photon splitting predominant regions. The emergent spectra were calculated in the cases of the line formation regions located in the middle and top of the atmosphere. We found that the location of the line formation regions affect the depth of the line. Furthermore, we calculated the emergent spectra in several optical depths of cyclotron resonant scattering predominant region and found that the cyclotron absorption lines become unclear in the higher resonant energies than 200 keV.

keywords: Cyclotron lines, photon splitting, super-strong magnetic field

1. Introduction

The study of neutron stars with extremely strong magnetic field has recently become active, because the observational data increases and physical phenomenon that can occur in such a strong magnetic field is very interesting. For example, soft gamma repeaters (SGRs) are considered to be ultra-magnetized neutron stars, $B > 10^{10}$ Tesla, so-called, "magnetars". SGR 1806-20 and 1900+14 have turned out to be X-ray pulsars. The magnetic field strength of these SGRs at the surface of the star had obtained to be 8×10^{10} T, assuming magnetic dipole braking and using the observed period P and period derivative \dot{P} .

A group of X-ray pulsars with periods around 6-12 seconds which are called anomalous X-ray pulsars (AXPs) are also considered to have super-strong magnetic field. They exhibit anomalous characteristics in comparison with the properties of known accreting X-ray pulsars. There is no sign of any companion. They are bright X-ray sources with luminosities $L_x \sim 10^{35} \text{ ergs}^{-1}$, steadily spinning down. Assuming that the spin-down of pulsars is a result of electromagnetic dipole radiation, AXPs have inferred surface dipole magnetic field strengths $B = (0.6 - 8) \times 10^{10}$ T.

Furthermore, about two dozen radio pulsars which have spin down fields greater than 10^9 T have discovered such as PSR 1509-58. Radio pul-

sars that have magnetic field larger than 10^9 T have become popular. Harding, Baring, and Gonthier (1997) found that photon splitting, or combined splitting and pair production, can explain the gamma ray spectrum of PSR 1509-58, where it inhibits emission above 1 MeV. They also found that photon splitting is important for γ -ray pulsars having $B_0 \geq 0.3B_c$.

Baring (1991, 1993, 1995) investigated the effect of photon splitting on the spectra formation of γ -ray bursts or SGR in details. He found that various effects of photon splitting reveal themselves on the spectra in ultra-magnetized atmospheres.

As to cyclotron lines, Kendziorra et al. (1994) found the feature of two absorption lines around 50 keV and 100 keV in the phase resolved TTM/HEXE spectra in the transient X-ray binary pulsar A0535+26, while Grove et al. (1995) found that the structure of an absorption feature near 110 keV in the phase averaged spectrum observed by OSSE. If it is due to cyclotron scattering at the first harmonic, the magnetic field required is about 10^9 T. Araya and Harding (1996a,b) also discussed the cyclotron resonant energy in pulsar A0535+26 by calculating the spectra in the two cases assuming an absorption feature to be the first and second harmonic line. They concluded that the observed 110 keV feature is more likely to be a first harmonic, based on the spectra calculated by their theoretical models. Maisack et al. (1997) studied the phase resolved spectrum of A0535+26 during a giant outburst. They found that a cyclotron resonance feature at 110 keV is required at every phase. They concluded that the feature at 110 keV is the first harmonic line, since there is no

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sign of a change in beam configuration at moderate optical depth.

Cyclotron lines directly provide a confirmation of the existence of a magnetic field and have been detected in the spectra of accreting pulsars and GRBs. The energy of cyclotron lines suggest the strength of the magnetic fields B to be $\sim 10^8$ T. A large number of numerical calculations (Nagel 1981; Meszaros, Nagel 1985; Wang et al. 1988, 1989; Lamb et al. 1989; Alexander et al. 1989; Lamb et al. 1990; Alexander, Meszaros 1991; Nishimura, Ebisuzaki 1992; Wang et al. 1993; Araya, Harding 1996a, 1996b; Alexander et al. 1996; Isenberg et al. 1998a, 1998b) regarding the cyclotron lines of pulsars and GRBs have been performed using the Monte-Carlo method or the Feautrier method. Araya, Harding (1999) constructed a numerical scheme for the production of relativistic theoretical cyclotron spectra from internally irradiated scattering atmospheres in the low-density /high field regime. They investigated the features of cyclotron lines in the atmospheres threaded by super-strong magnetic fields ($\approx 10^9$ T).

We calculated the radiative transfer in neutron star atmospheres comprised of the two kind of uniform density layer threaded by super-strong magnetic field ($\approx 10^9$ T), including both cyclotron resonant scattering and photon splitting. The relativistic effects in the Maxwell-Boltzmann distribution of the electron were ignored for the sake of simplicity, though we needed to solve the radiative-transfer equation while taking into account the relativistic effects for a plasma with temperatures of $kT > 50$ keV. We also ignored Compton scattering of the continuum component, since photon-splitting effects would be dominant in plasma density employed in the present calculation (Adler 1971). We will want to improve the problem described above in the future.

For neutron-star atmospheres with super-strong magnetic fields ($\approx 10^9$ T), we study the effect of the two kind of density layer on the spectra. Although the cyclotron resonant scattering depends strongly on the plasma density, photon splitting doesn't almost depend on it at the lower density we employed in this calculation (Bulik 1998). The structure of the cyclotron lines will, therefore, be affected considerably by photon splitting more than the case of the atmosphere with uniform density.

2. Calculation Method

Nishimura et al. (2000) found that the cyclotron absorption lines disappear due to the re-processing of photon splitting at the magnetic field strength corresponding to more than about

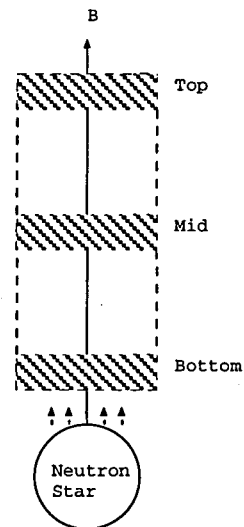


Figure 1 The structure of the atmospheres are employed in this paper. The shaded squares indicate the cyclotron scattering predominant layer and the dashed line show the whole atmosphere. The magnetic field is assumed to be perpendicular to the slab and constant.

~ 300 keV in the cyclotron energy in the atmosphere into which in the power-law-type spectra are injected. While cyclotron resonant scattering with an electron is impossible in plasma without the electron, photon splitting can occur if there is only a super-strong magnetic field near a quantum critical field. In Nishimura et al. (2000), the atmosphere was assumed to be unit uniform density and temperature plasma at the small region in space. In real atmosphere, the density, however, is not uniform and exists in large-scale space. Taking into account only higher density region is sufficient to calculate the cyclotron resonant scattering but we must include the lower density region in order to consider photon splitting. This fact will affect photon splitting which occurs over large-scale space on the cyclotron absorption lines in emergent spectra. In the present paper, we consider the atmosphere consisted of the higher density layer where cyclotron resonant scattering dominate and the lower density layer where photon splitting dominate. The layer that is dominated by cyclotron resonant scattering is assumed to be narrower region, 100-200 m, and one that is dominated by photon splitting is assumed to be large-scale region, 800-900 m; total height of the atmospheres is assumed to be 1 km. Thus, we investigate how the cyclotron absorption lines will be influenced by photon splitting in some strengths of the strong magnetic field. We calculate the emergent spectra by the method similar to Nishimura et al. (2000).

The magnetic photon splitting active, $\gamma \rightarrow \gamma\gamma$, are considered in large-scale layer threaded by uniform super-strong magnetic fields ($\approx 10^9$ T). The geometry which is employed in this calculation is depicted in Fig. 1. The shaded squares indicate the cyclotron resonant scattering predominant layer which has the higher density. The dashed square indicate the whole atmosphere which has the lower density. We consider three cases which the cyclotron resonant scattering predominant layer is localized in the most upper zone (Top), the middle zone (Middle), and the most lower zone (Bottom) of the atmosphere. Although the cyclotron resonant scattering is negligible in much lower density region, photon splitting is active in all region. Photon splitting occurs regardless of the existence of the plasma, if there is only super-strong magnetic field. We assumed the magnetic field strength to be constant for simplicity, while the magnetic field is not constant in real neutron star. In the present paper, the height of the atmosphere is assumed to be 10^3 m. In this case, the strength of the magnetic field decreases until about 75 percent of that of the surface magnetic field, supposed that it is a dipole field. The variation of the field strength will, therefore, have effect somewhat on the spectra. However, the feature of the structure of the cyclotron lines will be expected to be not significantly influenced by the effect of the variation of the field strength.

According to Baring(1991), the optical depth of photon splitting, τ_{sp} , for a radiation emission region of size R and extended to consider $B \geq B_c$ regimes is expressed by

$$\tau_{sp} = 37.2 \cdot R \cdot \left(\frac{B}{B_c}\right)^6 \left(\frac{\hbar\omega}{m_e c^2}\right)^5 (1 - \mu^2)^3 \text{ m}^{-1}. \quad (1)$$

From this equation, it is understood that this phenomenon can occur only in magnetic fields approaching the quantum critical value, $B_c = 4.41 \times 10^9$ T. Photon splitting is a phenomenon predicted by quantum electrodynamics (QED), and is forbidden in field-free regions by the charge symmetry of the theory [the Furry theorem (Furry 1937)]. The first calculations of the rate of photon splitting were made by Skobov (1959) and Minguzzi (1961).

Since a relativistic quantum treatment is necessary for the cyclotron resonant scattering in $B \sim B_c$, we use the cross section for relativistic electrons and $B \sim B_c$ by summing over the final spin state and averaging over the photon polarization (Harding, Daugherty 1991).

$$\sigma_{abs}^n(\theta) = \frac{\alpha\pi^2 \hbar^2 c^2}{E_n} \frac{\Gamma_n/2\pi}{(\omega - \omega_n)^2 + \Gamma_n^2/4} \frac{e^{-Z} Z^{n-1}}{(n-1)!}$$

$$\times [(1 + \cos^2\theta) + \frac{Z}{n} \sin^2\theta], \quad (2)$$

where

$$Z \equiv \frac{\omega^2 \sin^2\theta}{2m^2 B'} \quad (3)$$

and

$$E_n = (m^2 + \omega^2 \cos^2\theta + 2nB'm^2)^{1/2}. \quad (4)$$

The cyclotron energy is given by

$$\hbar\omega_n = mc^2[(1 + 2nB'\sin^2\theta)^{1/2} - 1]/\sin^2\theta. \quad (5)$$

We use the approximate form as the cross section for cyclotron resonant scattering,

$$\sigma_{sc}^n \simeq \frac{\sigma_{abs}^n}{L_n} \frac{E_n^2 \Gamma_n / 2\pi m^2}{(\omega + \omega^2 \sin^2\theta / 2m - nB'm)^2 + (E_n \Gamma_n / 2m)^2} \quad (6)$$

Here, L_n is Lorentz profile. This expression is a much more accurate approximation to resonant scattering deriving by replacing the Lorentz profiles in the absorption cross section (Harding, Daugherty 1991). According to Wang et al.(1988), the vacuum modes are dominant for first-harmonic scattering in optically thick media in the high-field/low-density limit where $w/\delta \ll 1$. A polarization-averaged cross section is employed, since this condition is satisfied in this paper.

On performing the calculations, a static, plane-parallel, isothermal atmosphere was assumed. The magnetic field direction was taken to be perpendicular to the surface of the plane-parallel atmosphere. The structure of atmospheres are illustrated in Fig.1 The radiative transfer equation is solved by using the Feautrier method (Mihalas 1978), we used the complete redistribution function and the cross section derived by Harding and Daugherty(1991) for cyclotron resonant scattering. The higher harmonics than the 3rd harmonic are also ignored. In the future, we need to improve these assumptions in order to investigate the depth of the lines in detail.

The transition rate to the ground state from the 2nd harmonic (Latal 1986) is calculated by

$$\Gamma_{n0} = 2\tau_0 \frac{n^n}{(n-1)!} \frac{B}{B_c} \left(1 + \frac{2nB}{B_c}\right)^{-3/2} I, \quad (7)$$

with

$$I = \int_0^1 dy \frac{(1-\xi)^{n-1}}{(1+\xi)^{n+1}} \left(2n \frac{B}{B_c} - \frac{1-\xi}{\xi}\right) \exp\left(-n \frac{1-\xi}{1+\xi}\right), \quad (8)$$

where

$$\xi = \left[\left(1 + 2n \frac{B}{B_c} y^2\right) / \left(1 + 2n \frac{B}{B_c}\right)\right]^{1/2}. \quad (9)$$

However, for the transition rates at the third harmonic, we used the non-relativistic expressions (Herold et al. 1982) for the sake of simplicity.

In these calculations we employed the same boundary conditions as Nishimura et al.(2000). The inner boundary condition is free, and the outer one assumes no incoming radiation from outside.

3. Results and Discussion

We calculated the emergent spectra through an atmosphere threaded by a super-strong magnetic field with a set of 30 frequencies and four angles. We investigated the effect of the photon splitting on the structures in the cyclotron line, assuming that the depth of the atmosphere is comparatively large, $R = 1 \times 10^3$ m. The atmosphere, however, is assumed to be comprised of the two kind of layer with different density. One layer is the dominance of cyclotron resonant scattering, $\rho = 10^{-3}$ kg m $^{-3}$, and another part of the atmosphere is the dominance of photon splitting, $\rho = 10^{-6}$ kg m $^{-3}$. The temperature is constant, $kT = \hbar\omega_B/4$ (Lamb et al. 1990).

Harding, Baring, and Gonthier (1997) calculated the spectra of escaping photons from a cascade above a neutron-star polar cap, including both photon splitting and pair production, by means of a Monte-Carlo simulation. They used a power-law distribution as the injected photons. They found that photon splitting is important for γ -ray pulsars having $B_0 \geq 0.3B_c$.

In the present paper, the injected flux is assumed to be a power-law of the photon energy, $I(x) = E^{-1.2}$. The photon index is in the spectra of the X-ray pulsar A 0535+26, in which the cyclotron line features were confirmed at around about 100 keV (Kendziorra et al. 1994; Grove et al. 1995).

3-1 The cyclotron line formation region is located in Middle and Top of the atmosphere

We consider the two cases which the cyclotron scattering regions are located in the middle (Middle) and the most upper zone (Top) of atmosphere. Since the total depth of atmosphere is same, the optical depth of photon splitting is same but the position of the cyclotron lines formation region in the atmosphere is different. That will significantly affect the depth of cyclotron absorption lines.

Figure 2 and 3 illustrate the emergent spectra in locating the cyclotron scattering region in the Middle and Top of atmosphere for the resonant energy, $\hbar\omega_B = 153$ keV. The feature of the spectra of two cases is comparatively similar than the

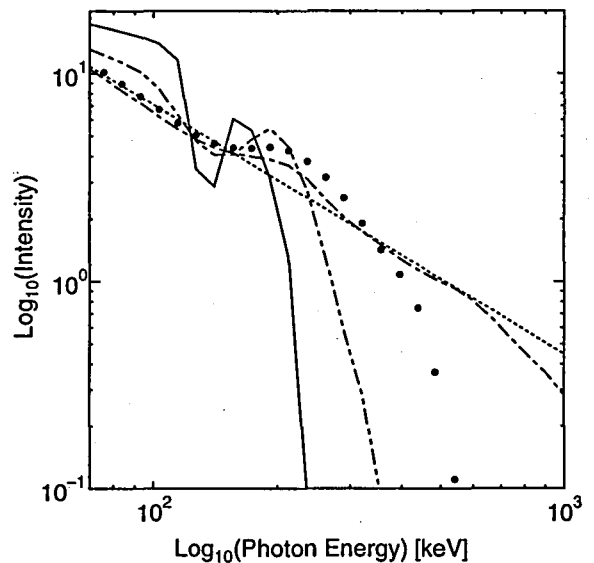


Figure 2 The emergent spectra in setting the cyclotron scattering dominant layer to the middle of atmosphere for the resonant energy, $\hbar\omega_B = 153$ keV. Here, the dotted line indicates the injected spectrum. The solid, two dot-dashed, bold dotted and dot-dashed curves show the spectra for $\mu = 0.1834, 0.5255, 0.7967,$ and 0.9603 , respectively.

case of setting it to the bottom of the atmosphere. The depth of the cyclotron lines in the first harmonic is different in the three cases, while the feature of the continuum spectra is similar. We see that the depth of the cyclotron absorption lines become deeper as the higher density region, in which cyclotron resonant scattering is dominant, is outer side of the atmosphere.

Figures 4 and 5 are same as figures 2 and 3, respectively, but for the resonant energy, $\hbar\omega_B = 200$ keV. In this resonant energy, the cyclotron absorption lines disappear even in the case of the Middle. In the case of the Top, the structure of the cyclotron lines, however, remain at the first harmonic. Figure 7 show the emergent spectra in atmosphere located the cyclotron line formation region in the most upper zone (Top) of atmosphere for the resonant energy, $\hbar\omega_B = 300$ keV. The cyclotron absorption lines all but disappear even in the case of the Top.

3-2 Some optical depths of the cyclotron line formation regions located in Bottom of the atmosphere

Figures 8, 9 and 10 show the emergent spectra in the case of Bottom, in which the cyclotron scattering predominant layer, 100 m, is located, for the resonant energy, $\hbar\omega_B = 100$ keV, 153 keV and 200 keV, respectively. In the resonant energy, $\hbar\omega_B = 100$ keV, the depth of the cyclotron lines

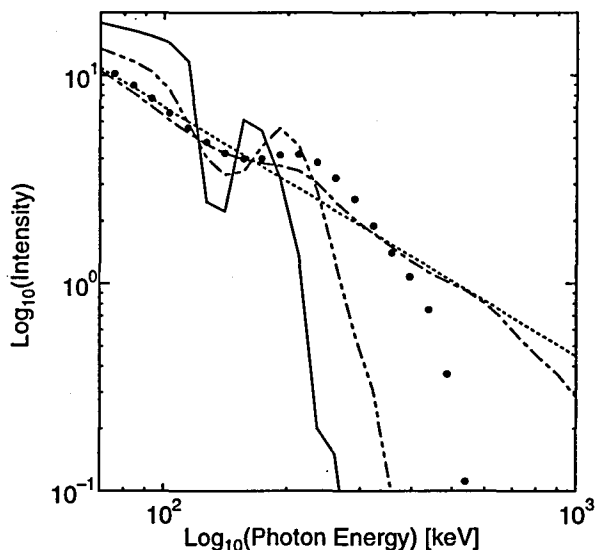


Figure 3 The emergent spectra in setting the cyclotron scattering dominant layer to the most upper zone (Top) of atmosphere for the resonant energy, $\hbar\omega_B = 153$ keV.

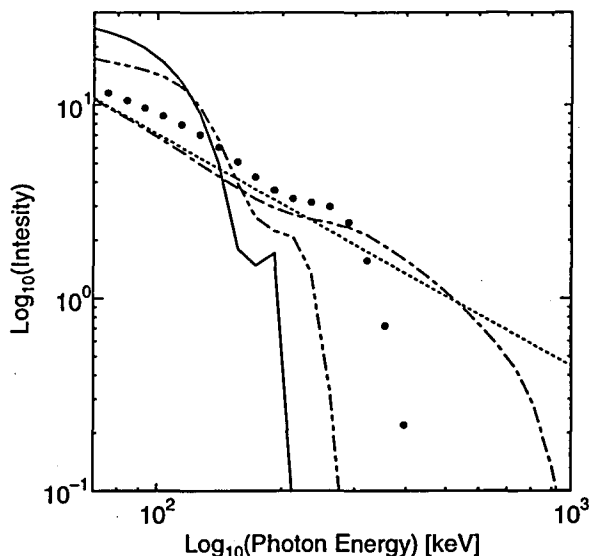


Figure 5 Same as figure 3, but for the resonant energy, $\hbar\omega_B = 200$ keV.

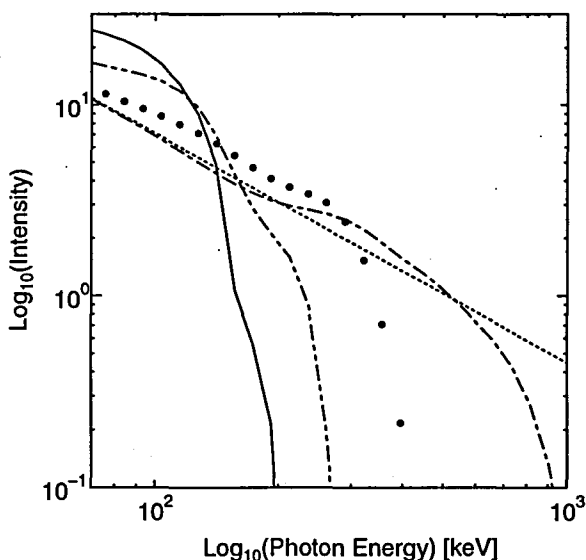


Figure 4 Same as figure 2, but for the resonant energy, $\hbar\omega_B = 200$ keV.

considerably change, compared to the cyclotron scattering predominant layer, 20 m. This is because the depth of the cyclotron scattering predominant layer is 100 m. The lines in the second and third harmonics are also clear in the direction perpendicular to the magnetic field. In the resonant energy, $\hbar\omega_B = 200$ keV, the lines in all harmonics, however, are unclear in all angles. This is because the photon of the cyclotron energy is converted to two photons of the lower energies by photon splitting. Furthermore, the cross section

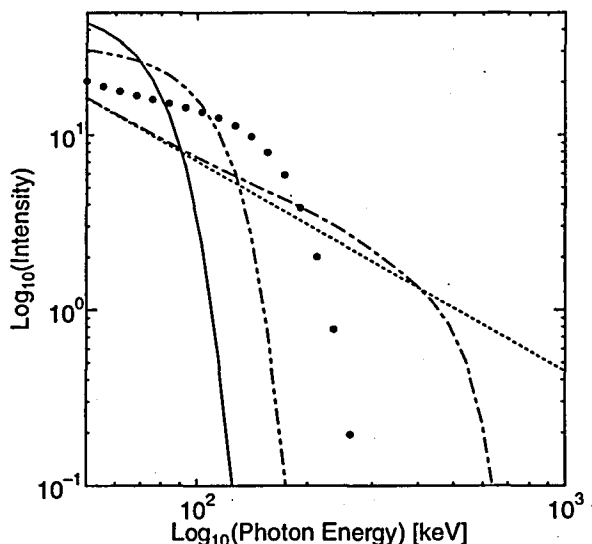


Figure 6 Same as figure 2, but for the resonant energy, $\hbar\omega_B = 300$ keV.

of the cyclotron resonant scattering also become very small in the angle parallel to the magnetic field in which that of photon splitting is small. Because of this, the cyclotron lines in all angles tend to disappear.

Figures 11, 12 and 13 are same as figures 8, 9 and 10, respectively, but for the atmosphere with the cyclotron scattering predominant layer whose depth is 200 m. In the resonant energy, $\hbar\omega_B = 100$ keV, the depth of the cyclotron absorption lines considerably become deep with the increase in the thickness of the cyclotron resonant scattering layer, since the cutoff energy

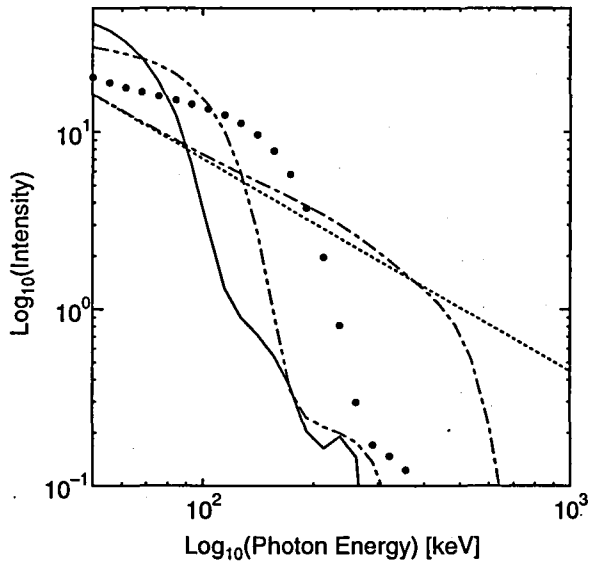


Figure 7 Same as figure 3, but for the resonant energy, $\hbar\omega_B = 300$ keV.

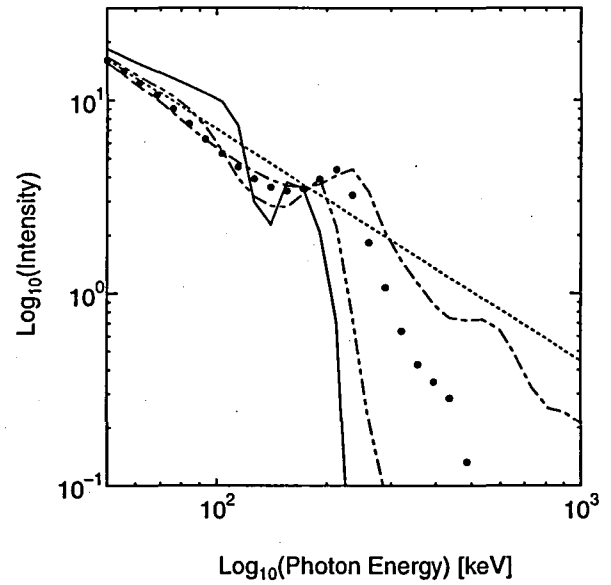


Figure 9 Same as figure 8, but for the resonant energy, $\hbar\omega_B = 153$ keV.

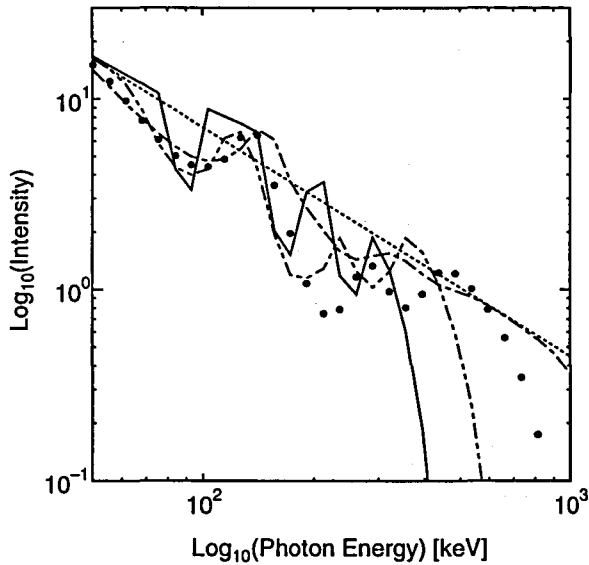


Figure 8 The emergent spectra in the case of Bottom with the cyclotron scattering predominant layer, 100 m, for the resonant energy, $\hbar\omega_B = 100$ keV.

due to photon splitting is higher than the energy of the third harmonic. In the resonant energy, $\hbar\omega_B = 153$ keV, the depth of the cyclotron absorption lines only in the first harmonic considerably change with the increase in the thickness of the cyclotron resonant scattering layer, since the cutoff energy due to photon splitting is higher than the energy of the first harmonic. Furthermore, the cyclotron absorption lines become noticeable in the angle where photon splitting is not active, since the optical depth of cyclotron reso-

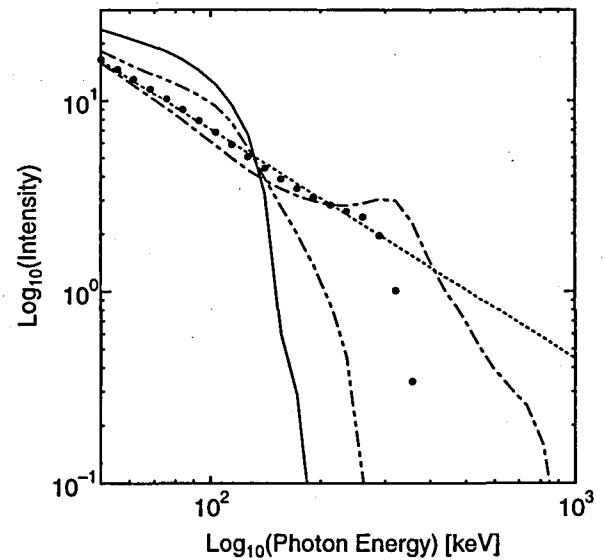


Figure 10 Same as figure 8, but for the resonant energy, $\hbar\omega_B = 200$ keV.

nant scattering is enough large to form the lines even in the angle parallel to the magnetic field. In the resonant energy, $\hbar\omega_B = 200$ keV, the depth of the cyclotron absorption lines will become no sufficient deep to detect the structure of it with the increase in the thickness of the cyclotron resonant scattering layer. This is because the cutoff energy due to photon splitting is lower than the resonant energy; the cyclotron absorption lines are unclear in the resonant energy, $\hbar\omega_B \geq 200$ keV.

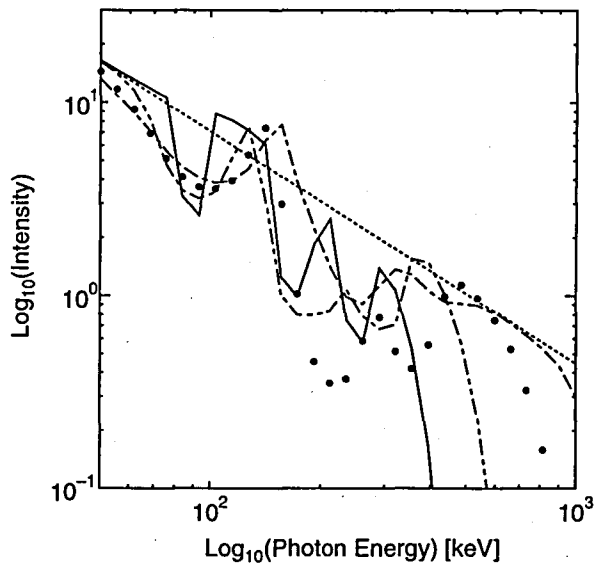


Figure 11 Same as figure 8, but for the cyclotron scattering predominant layer, 200 m.

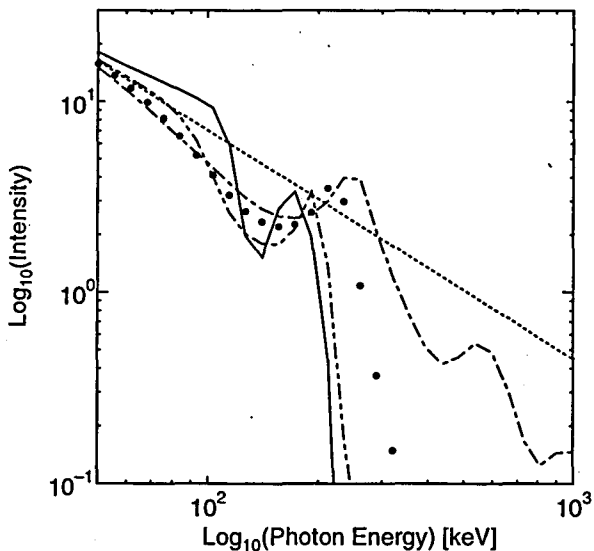


Figure 12 Same as figure 9, but for the cyclotron scattering predominant layer, 200 m.

4. Conclusions

In this calculation, we investigated the structures of the cyclotron lines reprocessed by photon splitting in strongly magnetized atmospheres with distinct line formation regions. First, the emergent spectra were calculated in atmospheres with line formation regions located at some sites (Top and Middle) of atmospheres. We found that this location influence somewhat the depth of the line. Second, we calculated the spectra in atmospheres with some depths of the line formation regions. Although the structures of the cyclotron

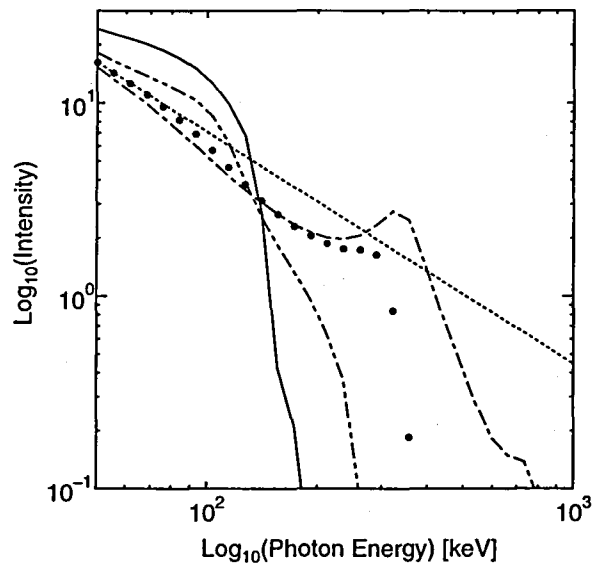


Figure 13 Same as figure 10, but for the cyclotron scattering predominant layer, 200 m.

lines is similar in both optical depths, the lines will be easier to be detected in the deeper optical depth.

In the conclusion, the cyclotron absorption lines will be unclear due to the reprocessing of photon splitting at the higher resonant energies than about 200 keV, independent of the depths of the atmospheres in this model where photon splitting occurs over 1 km, assuming the cyclotron lines formation region in the bottom region of the atmosphere. In the resonant energies from about 100 keV to 200 keV, the cyclotron absorption lines can be obscured by the reprocessing of photon splitting, when the column depth of the cyclotron formation region is such as was used in calculating the cyclotron lines in the spectrum of the gamma-ray burst GB880205 (Lamb et al. (1989) and Wang et al. (1989)). In the lower resonant energies than about 100 keV, the cyclotron lines will not be influenced by photon splitting.

Here, for the sake of simplicity, we assumed that the magnetic field strength is constant. This assumption will influence somewhat the optical depth of photon splitting and the energy of the cyclotron lines. We wish to consider the variation of the magnetic field in the future, by the assumption of the dipole magnetic field.

This calculation was performed by using the supercomputer provided by RIKEN.

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