A Study of the Crowning Method for the Type 3 Worm Gear*

(An Analysis of a Calculation Method for the Basic Thread Surface of the Hob)

Satoshi KISHI**, Muneharu MOROZUMI*** and Yoshitaroh YOSHIDA****

A precise analysis has been made on the crowning method for the type 3 worm gear. Firstly, the calculation method, which is to find the amount of the clearance between the thread surface of the worm and the modified tooth surface of the wheel in this crowning method, is obtained. This clearance is defined to be taken in the direction of circumference of the wheel and is calculated along the characteristic curve between the thread surface of the worm and the wheel tooth surface. Then, the design process for the hob, which cuts the modified wheel tooth surface so as to satisfy the given amount of the clearance, is clarified by using this calculation method. Theories in this analysis are verified by applying the calculation method and the design process to numerical examples of the type 3 worm gear. This calculation method and the design process are easily practicable by manufacturing engineers.

1. Introduction

In the theory of worm gearing, the thread surface of the worm meshes with the tooth surface of the wheel at a line of contact. But in practical operation, the edge contact is caused by the profile error or the setting error of the hob and the worm gear. Therefore, various studies of the crowning method for the wheel surface had been made to avoid the edge contact [1][2][3][4][5].

The purpose of this study is the completion of calculation method concerning the crowning method of the type 3 worm gear for manufacturing engineers on the personal computer. A precise analysis has been made on this crowning method. First of all, to find the clearance between the thread surface of the worm and the

* This work was presented at the Local Meeting of the Japan Society of Mechanical Engineers, held in Kanazawa (27th, June, 1986).
** Lecturer, Department of Mechanical Engineering.
*** Emeritus Professor, Shinshu University.
**** Assistant Manager, KOGANEI, LTD.
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modified tooth surface of the wheel, a calculation method is obtained. This clearance is defined to be taken in the direction of circumference of the wheel and is calculated along the characteristic curve between the surfaces of the worm thread and the wheel tooth. The calculation methods for the dimensions of the worm hob, which is used to form the wheel, and of the cone-shaped cutter, which is used to form the basic thread surface of the worm hob, are clarified. The principal notations are shown in Table 1.

Table 1. Nomenclature

α	: pressure angle of worm at arbitrary radius in axial section
α_a	: pressure angle of worm at pitch radius in axial section
l	: lead of worm
n	: number of threads on worm
P	: pitch of worm $(=l/n)$
h	: reduced pitch of worm $(=l/2\pi)$
r_c	: pitch radius of worm
r	: arbitrary radius of worm
β_c	: lead angle of worm at pitch radius $[=\tan^{-1}(l/2\pi r_c)]$
Z	: number of teeth of wheel
k	: gear ratio (Z/n)
R_c	: pitch radius of wheel $(=lZ/2\pi n=hk)$
R	: arbitrary radius of wheel
c	: center distance $(=r_c+R_c)$

2. Theoretical analysis

2-1 The thread surface of the type 3 worm

The thread surface of the type 3 worm established in JISB 1723 is chosen for that of the worm and the hob. The thread surface is expressed by the following equations on the o-xyz coordinate system fixed in the worm as shown in Fig. 1 [6]. The double sign indicates the order of the thread surfaces ① and ②.

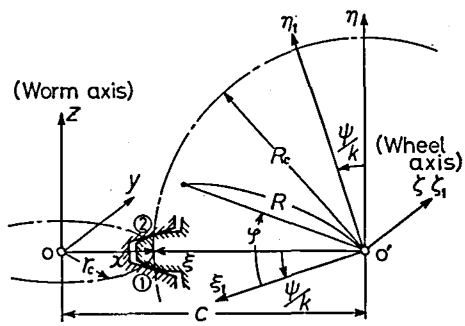


Fig 1. Coordinate relation between the worm and wheel

$$\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= -\rho\sin\Theta\sin\beta_c \mp\left[\frac{W}{2}-(\rho-\rho_c)\tan\alpha_c\right]\cos\beta_c + h(\lambda+\theta) \end{aligned}\right\} \quad (1)$$

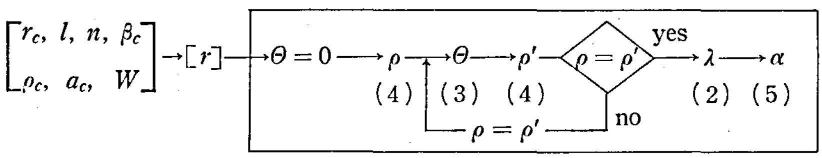
$$\tan\lambda = \frac{\rho\sin\Theta\cos\beta_c \mp\left[\frac{W}{2}-(\rho-\rho_c)\tan\alpha_c\right]\sin\beta_c}{a-\rho\cos\Theta} \quad (2)$$

where ρ : arbitrary radius of the cone-shaped cutter, ρ_c : pitch radius of the cutter, Θ: vectorial angle which is used to express the cutter surface, W: cutter width at pitch radius, α_c : cutter pressure angle, a : minimum distance between the worm axis and the cutter axis. The following equations exist among these dimensions and arbitrary radius r and pressure angle α. The calculation process (I) shows how to obtain the dimensions mentioned above.

$$\sin\Theta = \pm\Bigg[(a-r_c)\tan\alpha_c(r_c\tan\beta_c + a\cot\beta_c) - \Big[\rho\sec^2\alpha_c - \Big(\frac{W}{2}\cot\alpha_c + \rho_c\Big)\tan^2\alpha_c\Big]$$
$$\times\Big\{\Big[\rho\sec^2\alpha_c - \Big(\frac{W}{2}\cot\alpha_c + \rho_c\Big)\tan^2\alpha_c\Big]^2 + \tan^2\alpha_c(r_c\tan\beta_c + a\cot\beta_c)^2 - (a-r_c)^2\Big\}^{1/2}\Bigg]$$
$$\Big/\Big\{\Big[\rho\sec^2\alpha_c - \Big(\frac{W}{2}\cot\alpha_c + \rho_c\Big)\tan^2\alpha_c\Big]^2 + \tan^2\alpha_c(r_c\tan\beta_c + a\cot\beta_c)^2\Big\} \tag{3}$$

$$\rho = \Bigg[a\cos\Theta + \sin\beta_c\Big(\frac{W}{2} + \rho_c\tan\alpha_c\Big)(\tan\alpha_c\ \sin\beta_c \pm \cos\beta_c\sin\Theta)$$
$$-\Big\{r^2[(\tan\alpha_c\sin\beta_c \pm \cos\beta_c\sin\Theta)^2 + \cos^2\Theta] - \Big[\sin\beta_c\cos\Theta\Big(\frac{W}{2} + \rho_c\tan\alpha_c\Big)$$
$$-a(\tan\alpha_c\ \sin\beta_c \pm \cos\beta_c\ \sin\Theta)\Big]^2\Big\}^{1/2}\Bigg]\Big/[\cos^2\Theta + (\tan\alpha_c\ \sin\beta_c \pm \cos\beta_c\ \sin\Theta)^2] \tag{4}$$

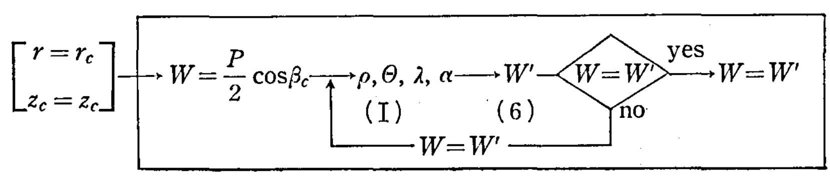
$$\tan\alpha = \mp\frac{dz}{dx} = \mp\Big(\frac{\partial z}{\partial\Theta} + \frac{\partial z}{\partial\rho}\frac{d\rho}{d\Theta} + \frac{\partial z}{\partial\lambda}\frac{d\lambda}{d\Theta}\Big)\frac{d\Theta}{dr}$$
$$= \mp\{r\cos^2\alpha_c\ \sec^2\Theta[(a-r_c)\sin\Theta \mp (a\cot\beta_c + r_c\tan\beta_c)\tan\alpha_c]$$
$$\times[(\sin\Theta\ \sin\beta_c \mp \tan\alpha_c\ \cos\beta_c)(a-\rho\cos\Theta) - r_c\tan\beta_c(\tan\lambda\ \cos\Theta + \sin\Theta\ \cos\beta_c$$
$$\pm\tan\alpha_c\ \sin\beta_c)\cos^2\lambda] + r\rho[\cos\Theta\ \sin\beta_c(a-\rho\cos\Theta) + r_c\tan\beta_c\ \cos\Theta\cos^2\lambda(\tan\lambda\ \tan\Theta$$
$$-\cos\beta_c)]\}\Bigg(\Big[(a-\rho\cos\Theta)\cos\Theta - \Big\{\rho\sin\Theta\ \cos\beta_c \mp \Big[\frac{W}{2} - (\rho-\rho_c)\tan\alpha_c\Big]\sin\beta_c\Big\}$$
$$\times(\sin\Theta\ \cos\beta_c \pm \tan\alpha_c\ \sin\beta_c)\Big]\cos^2\alpha_c\ \sec^2\Theta$$
$$\times[(a-r_c)\sin\Theta \mp (a\cot\beta_c + r_c\tan\beta_c)\tan\alpha_c] - (a-\rho\cos\Theta)\rho\sin\Theta$$
$$-\Big\{\rho\sin\Theta\ \cos\beta_c \mp \Big[\frac{W}{2} - (\rho-\rho_c)\tan\alpha_c\Big]\sin\beta_c\Big\}\rho\cos\Theta\ \cos\beta_c\Bigg)^{-1} \times (a-\rho\cos\Theta)^{-1} \tag{5}$$



(I)

In practical production of the worm, the value of the cutter width W is often taken as $W = (P/2)\cos\beta_c$. Then the groove width at pitch radius on axial section of the generated worm becomes slightly smaller than P/2[6]. Conversely, when the value of the groove width $2|z_c|$ is given, the correct value of the cutter width will be obtained by the following equation and the calculation process. From eq. (1), we obtain

$$W = 2[\mp(z_c + \rho\sin\Theta\ \sin\beta_c - h\lambda)\sec\beta_c + (\rho - \rho_c)\tan\alpha_c] \tag{6}$$



(II)

2-2 The tooth surface of the wheel and the characteristic curve

When the thread surface of the worm, which is expressed by Eq. (1), rotates in the positive direction by ψ, the worm does not move in the direction of its own axis, therefore the following equation is obtained.

$$\left.\begin{aligned} x &= r\cos(\theta+\psi) \\ y &= r\sin(\theta+\psi) \\ z &= -\rho\sin\Theta\sin\beta_c \\ &\quad \mp\left[\frac{W}{2}-(\rho-\rho_c)\tan\alpha_c\right]\cos\beta_c + h(\lambda+\theta) \end{aligned}\right\} \qquad (7)$$

The wheel rotates in the negative direction by ψ/k as shown in Fig. 1. Therefore the thread surface of the worm is expressed by the following equations as a family of curved surfaces on o'-$\xi_1\eta_1\zeta_1$ coordinate system fixed in the wheel, as shown in Fig. 1.

$$\left.\begin{aligned} \xi_1 &= [c-r\cos(\theta+\psi)]\cos\frac{\psi}{k} \\ &\quad -\left\{-\rho\sin\Theta\sin\beta_c\mp\left[\frac{W}{2}-(\rho-\rho_c)\tan\alpha_c\right]\cos\beta_c \right. \\ &\quad \left. + h(\lambda+\theta)\right\}\sin\frac{\psi}{k} \\ \eta_1 &= [c-r\cos(\theta+\psi)]\sin\frac{\psi}{k} \\ &\quad +\left\{-\rho\sin\Theta\sin\beta_c\mp\left[\frac{W}{2}-(\rho-\rho_c)\tan\alpha_c\right]\cos\beta_c \right. \\ &\quad \left. + h(\lambda+\theta)\right\}\cos\frac{\psi}{k} \\ \zeta_1 &= r\sin(\theta+\psi) \end{aligned}\right\} \qquad (8)$$

The tooth surface of the wheel is generated as the envelope of such a family of curved surfaces as in eq. (8). The condition for obtaining the envelope is as follows;

$$\begin{vmatrix} \partial\xi_1/\partial r & \partial\xi_1/\partial\theta & \partial\xi_1/\partial\psi \\ \partial\eta_1/\partial r & \partial\eta_1/\partial\theta & \partial\eta_1/\partial\psi \\ \partial\zeta_1/\partial r & \partial\zeta_1/\partial\theta & \partial\zeta_1/\partial\psi \end{vmatrix} = 0$$

By analyzing the above Jacobian, the following equation is obtained.

$$\begin{aligned} & r[r_c - r\cos(\theta+\psi)] \\ & +\left\{-\rho\sin\Theta\sin\beta_c\mp\left[\frac{W}{2}-(\rho-\rho_c)\tan\alpha_c\right]\cos\beta_c + h(\lambda+\theta)\right\} \\ & \qquad \times[h\sin(\theta+\psi)\pm r\tan\alpha\cos(\theta+\psi)] = 0 \end{aligned} \qquad (9)$$

Let $\theta+\psi=\gamma$ and A and B as follows.

$$\left.\begin{aligned} A &= c - r\cos\gamma \\ B &= -\rho\sin\Theta\,\sin\beta_c \\ &\quad \mp\left[\frac{W}{2}-(\rho-\rho_c)\tan\alpha_c\right]\cos\beta_c + h(\lambda+\gamma-\psi) \end{aligned}\right\} \tag{10}$$

From eq. (9) and (10), the following equations are obtained.

$$\begin{aligned} \psi &= \frac{1}{h}\left\{\frac{r(r_c - r\cos\gamma)}{h\sin\gamma \pm r\tan\alpha\,\cos\gamma} - \rho\sin\Theta\,\sin\beta_c \right. \\ &\qquad \left. \mp\left[\frac{W}{2}-(\rho-\rho_c)\tan\alpha_c\right]\cos\beta_c\right\} + (\lambda+\gamma) \end{aligned} \tag{11}$$

$$\begin{aligned} r &= \frac{1}{2}\{(r_c\sec\gamma \pm B\tan\alpha) \\ &\qquad + [(r_c\sec\gamma \pm B\tan\alpha)^2 + 4Bh\tan\gamma]^{1/2}\} \end{aligned} \tag{12}$$

These eqs. (9), (11) and (12) are also the equation of the condition for the characteristic curve between the thread surface of the worm and the wheel tooth surface. By using eq. (8), the enveloped tooth surface of the wheel is expressed by the following equation.

$$\left.\begin{aligned} \xi_1 &= R\cos\varphi \\ \eta_1 &= R\sin\varphi \\ \zeta_1 &= r\sin\gamma \end{aligned}\right\} \tag{13}$$

where

$$R^2 = A^2 + B^2 \tag{14}$$

$$\tan\varphi = \frac{\eta_1}{\xi_1} = \frac{A\sin\dfrac{\psi}{k} + B\cos\dfrac{\psi}{k}}{A\cos\dfrac{\psi}{k} - B\sin\dfrac{\psi}{k}} \tag{15}$$

The tooth profile on a plane perpendicular to the wheel axis (any ζ_1 plane) is obtained by the following equation and the calculation process where R_b: root radius of the wheel, R_k: tip radius of the wheel.

$$\gamma = \sin^{-1}\left(\frac{\zeta_1}{r}\right) \tag{16}$$

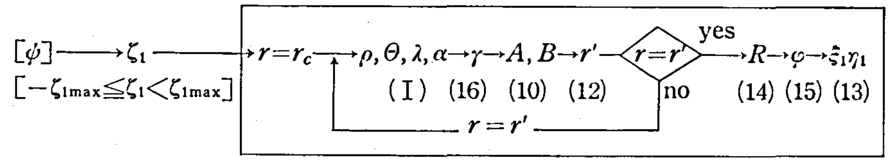
$$[\zeta_1] \longrightarrow [R] \longrightarrow r = r_c \longrightarrow \rho, \Theta, \lambda, a \rightarrow \gamma \rightarrow \psi \rightarrow A, B \rightarrow R' \rightarrow \langle R = R' \rangle \xrightarrow{\text{yes}} \varphi \rightarrow \xi_1\eta_1 \quad (\text{III})$$

$$[R_b \leqq R \leqq R_K] \qquad (\text{I}) \quad (16)\,(11) \quad (10) \quad (14) \quad \text{no} \quad (15)\,(13)$$

$$r$$

Infinite number of characteristic curves will exist corresponding to the rotational angle ψ of the worm. Therefore the characteristic curve can be decided as a single curve when the rotational angle ψ is determined.

The characteristic curve which passes through any point $[y=y(\zeta_1=\zeta_1),\ r=r]$ is obtained as follows. Firstly, by substituting the given value of r into the calculation process (I), ρ, Θ, λ and α are obtained. Then γ is obtained from Eq.(16). And by substituting these values into Eq.(11), ψ is obtained. By using this value of ψ, the characteristic curve is determined by the following calculation process where the value of ζ_{1max} is equal to one half of the face width of the wheel or is equal to the value of $r_c \tan 30°$.



(IV)

2-3 Growning method of the tooth surface of the wheel and the calculation process

In the crowning method of this study, the thread surface of the worm is not identical to the basic thread surface of the hob. The suffix $_w$ is used for the notations which represent the dimensions of the worm and the suffix $_h$ for that of the hob.

A crowning method for the type 1 worm gear had already been presented(1)(2). This method is as follows. The lead l_h of the basic thread surface of the hob is slightly smaller than that (l_w) of the worm. Similarly the pressure angle α_{ah} of the hob is slightly smaller than that (α_{aw}) of the worm.

Then the analysis of the calculation method, where the clearance on the pitch cylinder of the wheel in the crowning method can be found accurately and easily, has been made(3). If these hob and worm are used, a worm gearing will have the clearance in the direction of the face width of the wheel. The following equation is given for dimensions of hob and worm in this crowning method for type 1 worm gear.

$$l_w \cos\alpha_{aw} = l_h \cos\alpha_{ah} \qquad (17)$$

In the study of the crowning method for type 3 worm gear, the above mentioned crowning method and eq.(17) are also applied. And the given dimensions (l_w, r_{cw}, α_{aw}, l_h, r_{ch}, α_{ah}) of the worm and the hob satisfy the Eq.(17).

Now let's consider an imaginary tooth surface of the wheel generated by the worm and an actual tooth surface of the wheel generated by the hob. Then, an imaginary characteristic curve between the thread surface of the worm and the imaginary tooth surface of the wheel is considered.

The imaginary characteristic curve passes through the pitch point [$r=r_{cw}$] in the central plane of the imaginary wheel. In actual worm gearing between the worm and the actual wheel, the clearance in the direction of circumference of the wheel is calculated along the imaginary characteristic curve. To obtain the imaginary characteristic curve, it is necessary to determine the value $\psi=\psi_c$ by substituting the values $r=r_{cw}$ and $\zeta_1=0$ ($\gamma=0$) into the calculation process (I) and Eq. (11). The dimensions of transverse section of the actual and the imaginary wheels in the central plane of the wheel ($\zeta_1=0$) are expressed by the notations R_{cw}, φ_{wo}, ζ_{ho}, and the dimensions of the transverse section in any plane ($\zeta_1=\zeta_1$) of the wheel are expressed by the notations R_w, ζ_w, ζ_h. The geometrical relations of these dimensions viewed from the axial direction of the wheel are shown in Fig. 2.

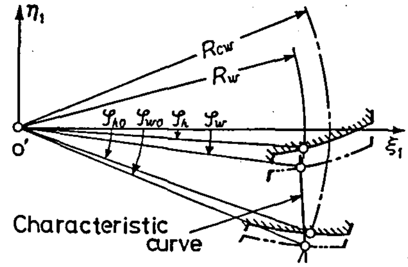


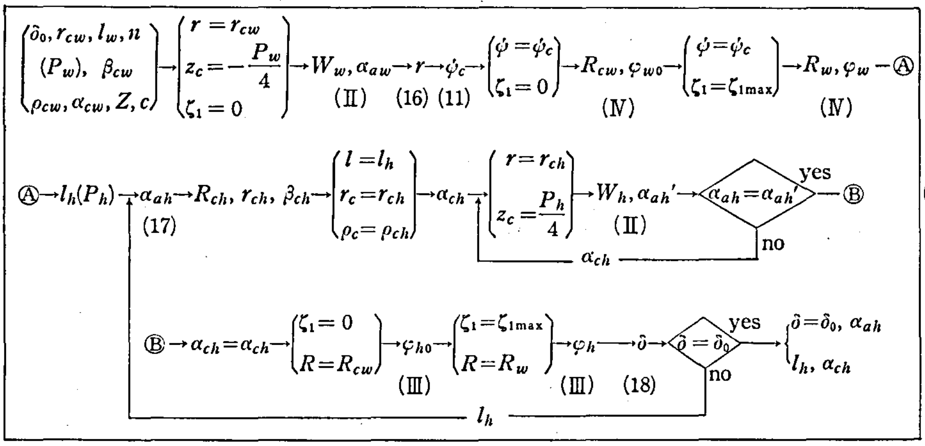
Fig 2. Schematic diagram of the characteristic curve and the transverse sections of the wheel

When the imaginary tooth surface of the wheel contacts with the actual tooth surface of the wheel at a point [$R=R_{cw}$ ($r=r_{cw}$)] in the central plane, the clearance δ between them in the direction of circumference of the wheel in any plane ($\zeta_1=\zeta_1$) is obtained by the following equation and the calculation process.

$$\delta=R_w[(\varphi_h-\varphi_{ho})-(\varphi_w-\varphi_{wo})] \cdots\cdots\cdots\cdots\cdots\cdots\cdots (18)$$

$$\begin{pmatrix} r_c=r_{cw} \\ l=l_w \\ \zeta_1=0 \\ r=r_{cw} \end{pmatrix} \xrightarrow[\text{(I) (11)}]{} \psi_c \longrightarrow \boxed{\begin{pmatrix} \psi=\psi_c \\ r_c=r_{cw} \\ l=l_w \end{pmatrix} \rightarrow \begin{pmatrix} \zeta_1=0 \\ \text{and} \\ \zeta_1=\zeta_1 \end{pmatrix} \underset{\text{(IV)}}{\rightarrow} \begin{matrix} R_{cw},\ \varphi_{wo} \\ \text{and} \\ R_w,\ \varphi_w \end{matrix} \rightarrow \begin{pmatrix} r_c=r_{ch} \\ l=l_h \end{pmatrix} \rightarrow \begin{pmatrix} R=R_{cw} \\ \text{and} \\ R=R_w \end{pmatrix} \underset{\text{(III)}}{\rightarrow} \begin{matrix} \varphi_{ho} \\ \text{and} \\ \varphi_h \end{matrix} \underset{(18)}{\rightarrow} \delta} \quad \text{(V)}$$

For the given dimensions of the worm, consider the imaginary tooth surface of the wheel formed imaginarily by the worm. Then, the imaginary characteristic curve which passes through the pitch point [$R=R_{cw}$] of the wheel in the central plane ($\zeta_1=0$) is considered. When the required clearance δ_0 on the plane ($\zeta_1=\zeta_{1max}$), which is taken in the direction of circumference along the imaginary characteristic curve, are given, the dimensions of the hob which satisfies the required δ_0 and the dimensions of the cone-shaped cutter are obtained by the following calculation process. Where the groove width at pitch radius in the axial section of the worm and the basic thread surface of the hob are determined as $P_w/2$, or $P_h/2$ respectively.



(VI)

3. Numerical example

Now the values of the dimensions of the worm and the wheel are chosen as follows: $r_{cw}=26.150$mm, $l_w=25.950$mm, $n=2$, $Z=77$, $P_w=12.975$mm, $\beta_c=8°58'30''$. The values of the dimensions of the cone-shaped cutter are chosen as follows: $\rho_{cw}=195$mm, $\alpha_{cw}=20°$. Then the dimensions of the imaginary wheel are as follows: $R_{cw}=159.008$mm, $c=185.158$ mm. Let required values be $\delta_0=0.055$mm at $\zeta_{1\max}=r_{cw}\tan 30°=15.098$mm. By substituting these given values into the calculation process (VI), the dimensions relating to the worm and the hob are obtained as shown in Table 2.

Table 2. Calculating results of dimensions of the worm and the hob

	Worm	Hob
l_w, l_h	(25.950)	25.832
r_{cw}, r_{ch}	(26.150)	26.870
n	(2)	(2)
P_w, P_h	12.975	12.916
β_{cw}, β_{ch}	8°58'30"	8°41'57"
α_{aw}, α_{ah}	20°12'26"	19°29'10"
ρ_{cw}, ρ_{ch}	(195)	(195)
α_{cw}, α_{ch}	(20°)	19°17'52"
W_w, W_h	6.409	6.384
δ_0		(0.055)
Z	(77)	(77)
R_{cw}, R_{ch}	159.008	158.287
c	185.158	185.158
ψ_c	−45°	
φ_{w0}, φ_{h0}	−1°10'8"	−1°4'30"
R_w	158.785	
φ_w, φ_h	−0°57'47"	−0°50'56"

( ): indicates given dimensions

The circumferential clearances between the wheel and the worm along the several characteristic curves are obtained by using the calculation process (V). These results are shown in Fig. 3 in μm unit. Since the value of clearance at $\zeta_1=\zeta_{1\max}$ along the characteristic curve corresponding to rotational angle ψ_c is equal to $\delta_0=0.055$mm, the result satisfies the required value. So tooth bearings will be excellent in the practical worm gearing between the worm and the modified wheel. The purpose of this study of the crowning method for the type 3 worm gear is accomplished.

4. Conclusions

The calculation method obtaining the dimensions of the type 3 worm hob, which is used to form the modified tooth surface of the wheel, and the dimensions of the cone-shaped cutter, which is used to form the basic thread surface of the hob, is clarified. Theories in this analysis are verified by applying the calculation method to numerical examples of the type 3 worm gear. Since the clearance in the analysis of this crowning method is calculated along the characteristic curve between the wheel and the worm, the analyzing method of this study agrees well with the practical worm gearing. If the crowning method of this study is used, it is expected that the excellent tooth bearing will be obtained. Necessity of the precise analysis of the crowning method for the type 3 worm gear is suggested by the numerical difference between type 3 worm gear and type 1 worm gear. Since the calculation method of this crowning method is described in detail, it should be easily understood by the manufacturing engineers.

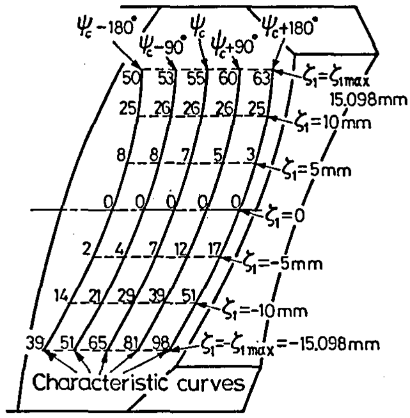


Fig. 3 Circumferential clearances between the wheel and worm along the several characteristic curves

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