はり問題の一解法

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A Solution of Beam Problems

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There are many cases in which discontinuity takes place in beam problems, so that this study is to try to find out a solution of these problems by the continuous expression of bending moment by trigonometrical series. This solution can be applied to many cases. To efficiently calculate, n in the series expression will be taken from 1 to 5, and the results are indicated in the tables. It is also shown in these tables that the comparison with these values and the values from former discontinuous expression ; these tables therefore show the range of application of this method.

1. 緒　　言

はり問題の解法では不連続性が伴うので三解級数を用いて bending moment を連続表示することによって一つの解法を試みた。この適用はきわめて平易な考えで理解されるが計算上で能率的に進めるため項数を第５項までにした場合どんな荷重や支持状態に適用できるかを従来の不連続表示における計算値と比較検討し構造物等への拡張発展について以下の計算例により説明する。

2. 計　算　式

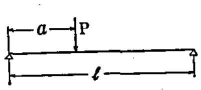


図　1

図１のような静定はりに一つの集中荷重がある場合 bending momeut を Fourier の sine の級数に展開する。

$$M=\sum An \sin\frac{n\pi x}{l}, \quad ここで\ A_n=\frac{2Pl}{n^2\pi^2}\sin\frac{n\pi a}{l}.$$

したがって bending moment (M), slope (θ), deflection (y), はつぎのようになる。

$$M=\frac{2Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi a}{l}\sin\frac{n\pi x}{l},$$

$$EI\theta=\frac{2Pl^2}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}\cos\frac{n\pi x}{l},$$

$$EIy=\frac{2Pl^3}{\pi^4}\sum\frac{1}{n^4}\sin\frac{n\pi a}{l}\sin\frac{n\pi x}{l}.$$

荷重状態が変っても重ね合せ法によって同じように展開される。

＊機械工学科

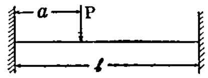


図 2

図2のような不静定はりにおいては不静定の moment を A*x*+B として bending moment を

$$M=\frac{2Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi a}{l}\sin\frac{n\pi x}{l}+Ax+B$$

とおき境界条件よりA・Bを決定する。

すなわち

$$M=\frac{2P}{\pi^2}\left\{l\sum\frac{1}{n^2}\sin\frac{n\pi a}{l}\sin\frac{n\pi x}{l}-\frac{2}{\pi}\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}\right.$$
$$\left.[l\{2+(-1)^n\}-3x\{1+(-1)^n\}]\right\},$$

$$EI\theta=\frac{2P}{\pi^3}\left\{\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}[l^2\cos\frac{n\pi x}{l}+2lx\{2+(-1)^n\}-3x^2\{1+(-1)^n\}-l^2\}]\right\},$$

$$EIy=\frac{2P}{\pi^3}\left\{\frac{l^3}{\pi}\sum\frac{1}{n^4}\sin\frac{n\pi a}{l}\sin\frac{n\pi x}{l}+\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}\right.$$
$$\left.[lx^2\{2+(-1)^n\}-x^3\{1+(-1)^n\}-l^2x]\right\}.$$

となり荷重状態が変つた場合は静定はりと同じく重ね合せによって展開する。

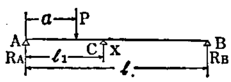


図 3

図3のような連続はりでは不静定量の反力 X を deflection の式を用いて Wilson の方法より容易に求めることができる。

$$EIy_1=\frac{2Pl^3}{\pi^4}\sum\frac{1}{n^4}\sin\frac{n\pi a}{l}\sin\frac{n\pi l_1}{l}\cdots\cdots P\text{によるC点のたわみ.}$$

$$EIy'_1=-\frac{2Xl^3}{\pi^4}\sum\frac{1}{n^4}\sin^2\left(\frac{n\pi l_1}{l}\right)\cdots\cdots\text{反力}X\text{によるC点のたわみ.}$$

$$\therefore X=\frac{-\sum\frac{1}{n^4}\sin\frac{n\pi a}{l}\sin\frac{n\pi l_1}{l}}{\sum\frac{1}{n^4}\sin^2\left(\frac{n\pi l_1}{l}\right)}\cdot P.$$

支点が数点ある場合も同じように deflection の式を作りその連立方程式の根が各支点の反力になる。

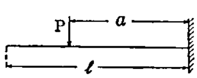


図 4

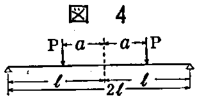


図 5

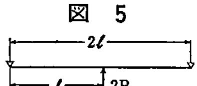


図 6

図4のような片持はりでは図5，図6のように分解して各々の bending moment を級数に展開して重ね合せる。

図5の bending moment は

$$M_1=\frac{8Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi x}{2l}\sin\frac{n\pi}{2}\cos\frac{n\pi a}{2l},$$

図6の bending Moment は

$$M_2=-\frac{8Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi x}{2l}\sin\frac{n\pi}{2},$$

したがって $M=M_1+M_2$ として

$$M=\frac{8Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi x}{2l}\sin\frac{n\pi}{2}\left(\cos\frac{n\pi a}{2l}-1\right),$$

$$EI\theta=\frac{16Pl^2}{\pi^3}\sum\frac{1}{n^3}\cos\frac{n\pi x}{2l}\sin\frac{n\pi}{2}\left(\cos\frac{n\pi a}{2l}-1\right),$$

$$EIy=\frac{32Pl^3}{\pi^4}\sum\frac{1}{n^4}\sin\frac{n\pi}{2}\left(\sin\frac{n\pi x}{2l}-\sin\frac{n\pi}{2}\right)\left(\cos\frac{n\pi a}{2l}-1\right)$$

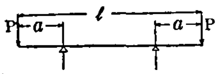


図 7

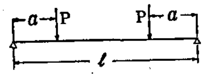


図 8

図７のような突出はりでは図８のような静定はりに集中荷重Pが対称に２個ある場合と同じで符号のみが反対になる。

$$M=-\frac{4Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi}{2}\cos\frac{(l-2a)n\pi}{2l}\sin\frac{n\pi x}{l},$$

$$EI\theta=-\frac{4Pl^2}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi}{2}\cos\frac{(l-2a)n\pi}{2l}\cos\frac{n\pi x}{l},$$

$$EIy=-\frac{4Pl^3}{\pi^4}\sum\frac{1}{n^4}\sin\frac{n\pi}{2}\cos\frac{(l-2a)n\pi}{2l}\sin\frac{n\pi x}{l}.$$

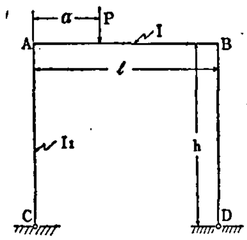


図 9

図９のような門型ラーメンにおいてはＡＢ材を一つのはりと考え荷重ＰによるＡ点の slope を級数で表わし

$EI\theta_1=\frac{2Pl^2}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}$，とし次に不静定 moment によるＡＢ材のＡ点の slope は $EI\theta_2=-\frac{Ml}{2}$，不静定 moment によるＡＣ材のＡ点の slope は $EI_1\theta_3=\frac{Mh}{3}$，　したがって $EI\theta_1+EI\theta_2=EI_1\theta_3$ より

$$M=\frac{4Pl}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}\left(\frac{1}{1+\frac{2}{3}\cdot\frac{h}{l}\cdot\frac{I}{I_1}}\right).$$

となる。

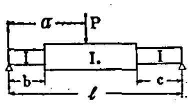


図 10

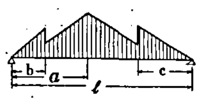


図 11

図10の変断面はりではその修正 moment を図11のようにして級数展開して修正 moment を求めると

$$M'=\sum\sin\frac{n\pi x}{l}\left[\frac{2P}{n^2\pi^2}\left\{\frac{Il}{I_o}\sin\frac{n\pi a}{l}+\frac{I_o-I}{I_o}\left((l-a)\sin\frac{n\pi b}{l}-a(-1)^n\sin\frac{n\pi c}{l}\right)\right\}\right.$$

$$\left.+\frac{2P}{n\pi}\cdot\frac{I_o-I}{I_o}\left\{\frac{a\cdot c}{l}(-1)^n\cos\frac{n\pi c}{l}-\frac{b(l-a)}{l}\cos\frac{n\pi b}{l}\right\}\right].$$

となる。

本研究における計算例の一般式をつぎに示す。

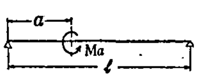


図 12

$$\left.\begin{aligned}M&=-\frac{2M_a}{\pi}\sum\frac{1}{n}\cos\frac{n\pi a}{l}\sin\frac{n\pi x}{l}\\EI\theta&=-\frac{2M_al}{\pi^2}\sum\frac{1}{n^2}\cos\frac{n\pi a}{l}\cos\frac{n\pi x}{l}\\EIy&=-\frac{2M_al^2}{\pi^3}\sum\frac{1}{n^3}\cos\frac{n\pi a}{l}\sin\frac{n\pi x}{l}\end{aligned}\right\}\quad\cdots\cdots\cdots(1)$$

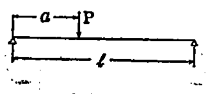


図 13

$$\left.\begin{aligned} M &= \frac{2Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi a}{l}\sin\frac{n\pi x}{l} \\ EI\theta &= \frac{2Pl^2}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}\cos\frac{n\pi x}{l} \\ EIy &= \frac{2Pl^3}{\pi^4}\sum\frac{1}{n^4}\sin\frac{n\pi a}{l}\sin\frac{n\pi x}{l} \end{aligned}\right\}\cdots\cdots(2)$$

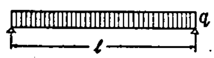


図 14

$$\left.\begin{aligned} M &= \frac{2ql^2}{\pi^3}\sum\frac{\{1-(-1)^n\}}{n^3}\sin\frac{n\pi x}{l} \\ EI\theta &= \frac{2ql^3}{\pi^4}\sum\frac{\{1-(-1)^n\}}{n^4}\cos\frac{n\pi x}{l} \\ EIy &= \frac{2ql^4}{\pi^5}\sum\frac{\{1-(-1)^n\}}{n^5}\sin\frac{n\pi x}{l} \end{aligned}\right\}\cdots\cdots(3)$$

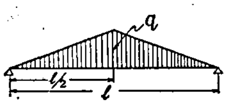


図 15

$$\left.\begin{aligned} M &= \frac{8ql^2}{\pi^4}\sum\frac{1}{n^4}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{l} \\ EI\theta &= \frac{8ql^3}{\pi^5}\sum\frac{1}{n^5}\sin\frac{n\pi}{2}\cos\frac{n\pi x}{l} \\ EIy &= \frac{8ql^4}{\pi^6}\sum\frac{1}{n^6}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{l} \end{aligned}\right\}\cdots\cdots(4)$$

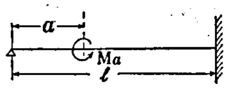


図 16

$$\left.\begin{aligned} M &= -\frac{2M_a}{\pi}\left[\sum\frac{1}{n}\cos\frac{n\pi a}{l}\{\sin\frac{n\pi x}{l}+\frac{3x}{n\pi l}(-1)^n\}\right] \\ EI\theta &= -\frac{M_a}{\pi^2}\left[\sum\frac{1}{n^2}\cos\frac{n\pi a}{l}\{2l\cos\frac{n\pi x}{l}-(\frac{3x^2}{l}-l)(-1)^n\}\right] \\ EIy &= -\frac{M_a}{\pi^2}\left[\sum\frac{1}{n^2}\cos\frac{n\pi a}{l}\{\frac{2l^2}{n\pi}\sin\frac{n\pi x}{l}-(\frac{x^3}{l}-lx)(-1)^n\}\right] \end{aligned}\right\}\cdots\cdots(5)$$

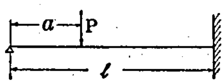


図 17

$$\left.\begin{aligned} M &= \frac{2P}{\pi^2}\left[\sum\frac{1}{n^2}\sin\frac{n\pi a}{l}\{l\sin\frac{n\pi x}{l}+\frac{3x}{n\pi}(-1)^n\}\right] \\ EI\theta &= \frac{P}{\pi^3}\left[\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}\{2l^2\cos\frac{n\pi x}{l}-(3x^2-l^2)(-1)^n\}\right] \\ EIy &= \frac{P}{\pi^3}\left[\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}\{\frac{2l^3}{n\pi}\sin\frac{n\pi x}{l}-(x^3-l^2x)(-1)^n\}\right] \end{aligned}\right\}\cdots\cdots(6)$$

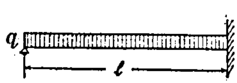


図 18

$$\left.\begin{aligned} M &= \frac{2ql}{\pi^3}\left[l\sum\frac{\{1-(-1)^n\}}{n^3}\sin\frac{n\pi x}{l}+\frac{3x}{\pi}\sum\frac{\{(-1)^n-1\}}{n^4}\right] \\ EI\theta &= \frac{ql}{\pi^4}\left[\sum\frac{\{1-(-1)^n\}}{n^4}\{2l^2\cos\frac{n\pi x}{l}-3x^2(-1)^n+l^2(-1)^n\}\right] \\ EIy &= \frac{ql}{\pi^4}\left[\sum\frac{\{1-(-1)^n\}}{n^4}\{\frac{2l^3}{n\pi}\sin\frac{n\pi x}{l}+(x^3-l^2x)\}\right] \end{aligned}\right\}\cdots\cdots(7)$$

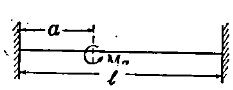


図 19

$$M = -\frac{2Ma}{\pi}\left[\sum\frac{1}{n}\cos\frac{n\pi a}{l}\sin\frac{n\pi x}{l}+\frac{2}{\pi}\sum\frac{1}{n^2}\cos\frac{n\pi a}{l}\right.$$
$$\left.\left(\frac{3x}{l}\{1+(-1)^n\}-\{2+(-1)^n\}\right)\right]$$

$$
\left.\begin{aligned}
EI\theta &= -\frac{2M_a}{\pi^2}\Big[\sum\frac{1}{n^2}\cos\frac{n\pi a}{l}\Big(l\cos\frac{n\pi x}{l}-\frac{3x^2}{l}\\
&\qquad \{1+(-1)^n\}+2x\{2+(-1)^n\}-l\Big)\Big]\\
EIy &= -\frac{2M_a}{\pi^2}\Big[\frac{l^2}{\pi}\sum\frac{1}{n^3}\cos\frac{n\pi a}{l}\sin\frac{n\pi x}{l}-\sum\frac{1}{n^2}\cos\frac{n\pi a}{l}\\
&\qquad \Big(\frac{x^3}{l}\{1+(-1)^n\}-x^2\{2+(-1)^n\}+lx\Big)\Big]
\end{aligned}\right\} \quad (8)
$$

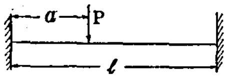


図 20

$$
\left.\begin{aligned}
M &= \frac{2P}{\pi^2}\Big[l\sum\frac{1}{n^2}\sin\frac{n\pi a}{l}\sin\frac{n\pi x}{l}-\frac{2}{\pi}\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}\\
&\qquad \Big(l\{2+(-1)^n\}-3x\{1+(-1)^n\}\Big)\Big]\\
EI\theta &= \frac{2P}{\pi^3}\Big[\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}\Big(l^2\cos\frac{n\pi x}{l}+2lx\{2+(-1)^n\}\\
&\qquad -3x^2\{1+(-1)^n\}-l^2\Big)\Big]\\
EIy &= \frac{2P}{\pi^3}\Big[\frac{l^3}{\pi}\sum\frac{1}{n^4}\sin\frac{n\pi a}{l}\sin\frac{n\pi x}{l}+\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}\\
&\qquad \Big(lx^2\{2+(-1)^n\}-x^3\{1+(-1)^n\}-l^2x\Big)\Big]
\end{aligned}\right\} \quad (9)
$$

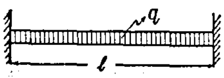


図 21

$$
\left.\begin{aligned}
M &= \frac{2ql}{\pi^3}\Big[l\sum\frac{\{1-(-1)^n\}}{n^3}\sin\frac{n\pi x}{l}-\frac{2}{\pi}\sum\frac{\{1-(-1)^n\}}{n^4}\\
&\qquad \Big(l\{2+(-1)^n\}-3x\{1+(-1)^n\}\Big)\Big]\\
EI\theta &= \frac{2ql}{\pi^4}\Big[\sum\frac{\{1-(-1)^n\}}{n^4}\Big(l^2\cos\frac{n\pi x}{l}+2lx\{2+(-1)^n\}\\
&\qquad -3x^2\{1+(-1)^n\}-l^2\Big)\Big]\\
EIy &= \frac{2ql}{\pi^4}\Big[\frac{l^3}{\pi}\sum\frac{\{1-(-1)^n\}}{n^5}\sin\frac{n\pi x}{l}+\sum\frac{\{1-(-1)^n\}}{n^4}\\
&\qquad \Big(lx^2\{2+(-1)^n\}-x^3\{1+(-1)^n\}-l^2x\Big)\Big]
\end{aligned}\right\} \quad (10)
$$

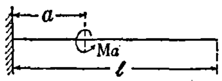


図 22

$$
\left.\begin{aligned}
M &= \frac{4M_a}{\pi}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n}\sin\frac{n\pi a}{2l}\cos\frac{n\pi x}{2l}\\
EI\theta &= -\frac{8M_a l}{\pi^2}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^2}\sin\frac{n\pi a}{2l}\sin\frac{n\pi x}{2l}\\
EIy &= \frac{16M_a l^2}{\pi^3}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^3}\sin\frac{n\pi a}{2l}\Big(\cos\frac{n\pi x}{2l}-1\Big)
\end{aligned}\right\} \quad (11)
$$

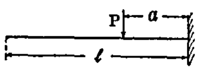


図 23

$$
\left.\begin{aligned}
M &= \frac{8Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi x}{2l}\sin\frac{n\pi}{2}\Big(\cos\frac{n\pi a}{2l}-1\Big)\\
EI\theta &= \frac{16Pl^2}{\pi^3}\sum\frac{1}{n^3}\cos\frac{n\pi x}{2l}\sin\frac{n\pi}{2}\Big(\cos\frac{n\pi a}{2l}-1\Big)\\
EIy &= \frac{32Pl^3}{\pi^4}\sum\frac{1}{n^4}\sin\frac{n\pi}{2}\Big(\sin\frac{n\pi x}{2l}-\sin\frac{n\pi}{2}\Big)\Big(\cos\frac{n\pi a}{2l}-1\Big)
\end{aligned}\right\} \quad (12)
$$

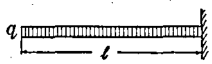


図 24

$$\left.\begin{aligned} M &= \frac{8ql^2}{\pi^2}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^2}\sin\frac{n\pi x}{2l}\left(\frac{2}{n\pi}-\sin\frac{n\pi}{2}\right) \\ EI\theta &= \frac{16ql^3}{\pi^3}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^3}\cos\frac{n\pi x}{2l}\left(\frac{2}{n\pi}-\sin\frac{n\pi}{2}\right) \\ EIy &= \frac{32ql^4}{\pi^4}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^4}\left(\sin\frac{n\pi x}{2l}-\sin\frac{n\pi}{2}\right)\left(\frac{2}{n\pi}-\sin\frac{n\pi}{2}\right) \end{aligned}\right\} \quad (13)$$

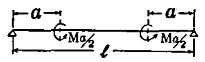


図 25

$$\left.\begin{aligned} M &= -\frac{2M_a}{\pi}\sum\frac{1}{n}\sin\frac{n\pi x}{l}\cos\frac{n\pi}{2}\cos\frac{(l-2a)n\pi}{2l} \\ EI\theta &= -\frac{2M_a l}{\pi^2}\sum\frac{1}{n^2}\cos\frac{n\pi x}{l}\cos\frac{n\pi}{2}\cos\frac{(l-2a)n\pi}{2l} \\ EIy &= -\frac{2M_a l^2}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi x}{l}\cos\frac{n\pi}{2}\,\mathrm{oos}\frac{(l-2a)n\pi}{2l} \end{aligned}\right\} \quad (14)$$

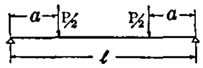


図 26

$$\left.\begin{aligned} M &= \frac{2Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi}{2}\cos\frac{(l-2a)n\pi}{2l}\sin\frac{n\pi x}{l} \\ EI\theta &= \frac{2Pl^2}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi}{2}\cos\frac{(l-2a)n\pi}{2l}\cos\frac{n\pi x}{l} \\ EIy &= \frac{2Pl^3}{\pi^4}\sum\frac{1}{n^4}\sin\frac{n\pi}{2}\cos\frac{(l-2a)n\pi}{2l}\sin\frac{n\pi x}{l} \end{aligned}\right\} \quad (15)$$

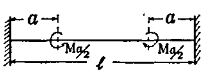


図 27

$$\left.\begin{aligned} M &= -\frac{2M_a}{\pi}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n}\sin\frac{n\pi}{2}\sin\frac{n\pi(l-2a)}{2l}\left(\sin\frac{n\pi x}{l}-\frac{2}{n\pi}\right) \\ EI\theta &= -\frac{2M_a}{\pi^2}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi(l-2a)}{2l}\left(l\cos\frac{n\pi x}{l}+2x-l\right) \\ EIy &= -\frac{2M_a}{\pi^2}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi(l-2a)}{2l}\left(\frac{l^2}{n\pi}\sin\frac{n\pi x}{l}+x^2-lx\right) \end{aligned}\right\} \quad (16)$$

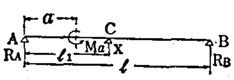


図 28

$$X = \frac{\pi M_a\sum\frac{1}{n^3}\cos\frac{n\pi a}{l}\sin\frac{n\pi l_1}{l}}{l\sum\frac{1}{n^4}\sin^2\left(\frac{n\pi l_1}{l}\right)}. \quad \cdots\cdots\cdots (17)$$

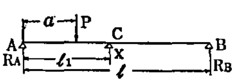


図 29

$$X = -\frac{\sum\frac{1}{n^4}\sin\frac{n\pi a}{l}\sin\frac{n\pi l_1}{l}}{\sum\frac{1}{n^4}\sin^2\left(\frac{n\pi l_1}{l}\right)}\cdot P. \quad \cdots\cdots\cdots (18)$$

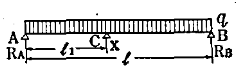


図 30

$$X = -\frac{ql\sum\frac{\{1-(-1)^n\}}{n^5}\sin\frac{n\pi l_1}{l}}{\pi\sum\frac{1}{n^4}\sin^2\left(\frac{n\pi l_1}{l}\right)}, \quad \cdots\cdots\cdots (19)$$

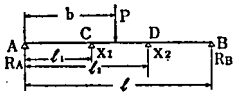


図 31

$$X_1=-\frac{\begin{vmatrix} k_1 & a_{12} \\ k_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}\cdot P\ ,\qquad X_2=-\frac{\begin{vmatrix} a_{11} & k_1 \\ a_{21} & k_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}\cdot P \ \cdots\cdots (20)$$

ここで $k_1=\sum\frac{1}{n^4}\sin\frac{n\pi b}{l}\sin\frac{n\pi l_1}{l}$, $k_2=\sum\frac{1}{n^4}\sin\frac{n\pi b}{l}\sin\frac{n\pi l_2}{l}$,

$$a_{11}=\sum\frac{1}{n^4}\sin^2\left(\frac{n\pi l_1}{l}\right)\ ,\qquad a_{22}=\sum\frac{1}{n^4}\sin^2\left(\frac{n\pi l_2}{l}\right)\ ,$$

$$a_{12}=a_{21}=\sum\frac{1}{n^4}\sin\frac{n\pi l_1}{l}\sin\frac{n\pi l_2}{l}\ .$$

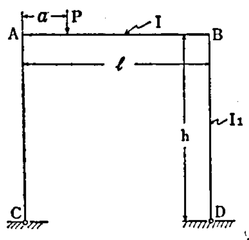


図 32

$$M=\frac{4Pl}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi a}{l}\left(\frac{1}{1+\frac{2}{3}\cdot\frac{h}{l}\cdot\frac{I}{I_1}}\right)\ \cdots\cdots (21)$$

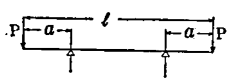


図 33

$$\left.\begin{aligned} M &=-\frac{4Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi}{2}\cos\frac{(l-2a)n\pi}{2l}\sin\frac{n\pi x}{l}\ , \\ EI\theta &=-\frac{4Pl^2}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi}{2}\cos\frac{(l-2a)n\pi}{2l}\cos\frac{n\pi x}{l}\ , \\ EIy &=-\frac{4Pl^3}{\pi^4}\sum\frac{1}{n^4}\sin\frac{n\pi}{2}\cos\frac{(l-2a)n\pi}{2l}\sin\frac{n\pi x}{l}\ . \end{aligned}\right\}(22)$$

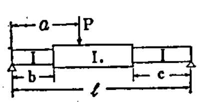


図 34

$$\left.\begin{aligned} M=\sum\Big[&\frac{2P}{n^2\pi^2}\left\{\frac{Il}{I_o}\sin\frac{n\pi a}{l}+\frac{I_o-I}{I_o}\Big((l-a)\sin\frac{n\pi b}{l}\right. \\ &\left.-a(-1)^n\sin\frac{n\pi c}{l}\Big)\right\}+\frac{2P}{n\pi}\cdot\frac{I_o-I}{I_o}\left\{\frac{a\cdot c}{l}(-1)^n\cos\frac{n\pi c}{l}\right. \\ &\left.-\frac{(l-a)b}{l}\cos\frac{n\pi b}{l}\right\}\Big]\sin\frac{n\pi x}{l} \end{aligned}\right\}(23)$$

3. 数値計算（従来の計算値との比較）

nは第5項まで

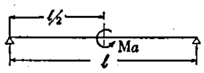


図 35

計算式

$$M_{x=l/4} = -\frac{2M_a}{\pi}\sum\frac{1}{n}\cos\frac{n\pi}{2}\sin\frac{n\pi}{4}$$

$$EI\theta_{x=l/4} = -\frac{2M_a l}{\pi^2}\sum\frac{1}{n^2}\cos\frac{n\pi}{2}\cos\frac{n\pi}{4}$$

$$EIY_{x=l/4} = -\frac{2M_a l^2}{\pi^3}\sum\frac{1}{n^3}\cos\frac{n\pi}{2}\sin\frac{n\pi}{4}$$

従来の計算値	$M_{x=l/4}=$ 0.250000M_a	$EI\theta_{x=l/4}=$ 0.010416$M_a l$	$EIY_{x=l/4}=$ 0.007812$M_a l^2$
級数	0.265763M_a	0.010621$M_a l$	0.007816$M_a l^2$
誤差	6.30%	0.19%	0.05%
精度	△	○	○

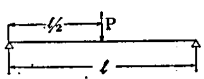


図 36

計算式

$$M_{x=l/4} = \frac{2Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi}{4}$$

$$EI\theta_{x=l/4} = \frac{2Pl^2}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi}{2}\cos\frac{n\pi}{4}$$

$$EIY_{x=l/4} = \frac{2Pl^3}{\pi^4}\sum\frac{1}{n^4}\sin\frac{n\pi}{2}\sin\frac{n\pi}{4}$$

従来の計算値	$M_{x=l/4}=$ 0.125000Pl	$EI\theta_{x=l/4}=$ 0.046875Pl^2	$EIY_{x=l/4}=$ 0.014322Pl^3
級数	0.126330Pl	0.046928Pl^2	0.014324Pl^3
誤差	1.06%	0.11%	0.01%
精度	○	○	○

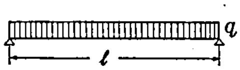


図 37

計算式

$$M_{x=l/4} = \frac{2ql^2}{\pi^3}\sum\frac{\{1-(-1)^n\}}{n^3}\sin\frac{n\pi}{4}$$

$$EI\theta_{x=l/4} = \frac{2ql^3}{\pi^4}\sum\frac{\{1-(-1)^n\}}{n^4}\cos\frac{n\pi}{4}$$

$$EIY_{x=l/4} = \frac{2ql^4}{\pi^5}\sum\frac{\{1-(-1)^n\}}{n^5}\sin\frac{n\pi}{4}$$

従来の計算値	$M_{x=l/4}=$ 0.093750ql^2	$EI\theta_{x=l/4}=$ 0.028645ql^3	$EIY_{x=l/4}=$ 0.009277ql^4
級数	0.093728ql^2	0.028640ql^3	0.009278ql^4
誤差	0.02%	0.02%	0.01%
精度	○	○	○

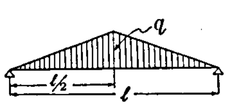


図 38

計算式

$$M_{x=l/4} = \frac{8ql^2}{\pi^4}\sum\frac{1}{n^4}\sin\frac{n\pi}{2}\sin\frac{n\pi}{4}$$

$$EI\theta_{x=l/4} = \frac{8ql^3}{\pi^5}\sum\frac{1}{n^5}\sin\frac{n\pi}{2}\cos\frac{n\pi}{4}$$

$$EIY_{x=l/4} = \frac{8ql^4}{\pi^6}\sum\frac{1}{n^6}\sin\frac{n\pi}{2}\sin\frac{n\pi}{4}$$

従来の計算値	$M_{x=l/4}=$ 0.057291ql^2	$EI\theta_{x=l/4}=$ 0.028645ql^3	$EIY_{x=l/4}=$ 0.005875ql^4
級数	0.057300ql^2	0.028640ql^3	0.005875ql^4

誤差	0.02%	0.01%	0 %
精度	○	○	○

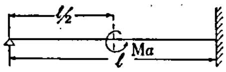


図 39

計算式	$M_{x=l/4} = -\frac{2M_a}{\pi}\sum\frac{1}{n}\cos\frac{n\pi}{2}\left(\sin\frac{n\pi}{4}+\frac{3}{4n\pi}\right)$ $EI\theta_{x=l/4} = -\frac{M_a l}{\pi^2}\sum\frac{1}{n^2}\cos\frac{n\pi}{2}\left(2\cos\frac{n\pi}{4}+\frac{3}{16}\right)$ $EIY_{x=l/4} = -\frac{M_a l^2}{\pi^2}\sum\frac{1}{n^2}\cos\frac{n\pi}{2}\left(\frac{2}{n\pi}\sin\frac{n\pi}{4}+\frac{15}{64}\right)$		
従来の計算値	$M_{x=l/4}=$ $0.281250M_a$	$EI\theta_{x=l/4}=$ $0.027343M_a l$	$EIY_{x=l/4}=$ $0.012695M_a l^2$
級数	$0.307731M_a$	$0.026758M_a l$	$0.012807M_a l^2$
誤差	9.40%	2.10%	1.50%
精度	×	○	○

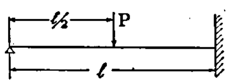


図 40

計算式	$M_{x=l/4} = \frac{2Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi}{2}\left\{\sin\frac{n\pi}{4}+\frac{3}{4n\pi}(-1)^n\right\}$ $EI\theta_{x=l/4} = \frac{Pl^2}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi}{2}\left\{2\cos\frac{n\pi}{4}+\frac{13}{16}(-1)^n\right\}$ $EIY_{x=l/4} = \frac{Pl^3}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi}{2}\left\{\frac{2}{n\pi}\sin\frac{n\pi}{4}+\frac{15}{64}(-1)^n\right\}$		
従来の計算値	$M_{x=l/4}=$ $0.078125Pl$	$EI\theta_{x=l/4}=$ $0.021484Pl^2$	$EIY_{x=l/4}=$ $0.006998Pl^3$
級数	$0.079433Pl$	$0.021461Pl^2$	$0.006996Pl^3$
誤差	1.67%	0.10%	0.02%
精度	○	○	○

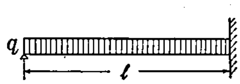


図 41

計算式	$M_{x=l/4} = \frac{2ql^2}{\pi^3}\sum\frac{\{1-(-1)^n\}}{n^3}\left(\sin\frac{n\pi}{4}-\frac{3}{4n\pi}\right)$ $EI\theta_{x=l/4} = \frac{ql^3}{\pi^4}\sum\frac{\{1-(-1)^n\}}{n^4}\left\{2\cos\frac{n\pi}{4}+\frac{13}{16}(-1)^n\right\}$ $EIY_{x=l/4} = \frac{ql^4}{\pi^4}\sum\frac{\{1-(-1)^n\}}{n^4}\left(\frac{2}{n\pi}\sin\frac{n\pi}{4}-\frac{15}{64}\right)$		
従来の計算値	$M_{x=l/4}=$ $0.062500ql^2$	$EI\theta_{x=l/4}=$ $0.011718ql^3$	$EIY_{x=l/4}=$ $0.004394ql^4$
級数	$0.064475ql^2$	$0.011724ql^3$	$0.004402ql^4$
誤差	3.10%	0.05%	2.40%
精度	○	○	○

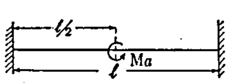


図 42

計算式	$M_{x=l/4} = -\frac{2M_a}{\pi}\left[\sum\frac{1}{n}\cos\frac{n\pi}{2}\sin\frac{n\pi}{4}-\frac{1}{2\pi}\sum\frac{1}{n^2}\cos\frac{n\pi}{2}\{5-(-1)^n\}\right]$ $EI\theta_{x=l/4} = -\frac{2M_a l}{\pi^2}\sum\frac{1}{n^2}\cos\frac{n\pi}{2}\left\{\cos\frac{n\pi}{4}-\frac{3}{16}+\frac{5}{16}(-1)^n\right\}$ $EIY_{x=l/4} = -\frac{2M_a l^2}{\pi^2}\left[\frac{1}{\pi}\sum\frac{1}{n^3}\cos\frac{n\pi}{2}\sin\frac{n\pi}{4}-\sum\frac{1}{n^2}\cos\frac{n\pi}{2}\left\{\frac{9}{64}-\frac{3}{64}(-1)^n\right\}\right]$		

従来の計算値	$M_{x=l/4}=0.125000M_a$	$EI\theta_{x=l/4}=0.015625M_al$	$EIY_{x=l/4}=0.003906M_al^2$
級数	$0.148415M_a$	$0.014807M_al$	$0.003846M_al^2$
誤差	18.70%	5.20%	1.50%
精度	×	△	○

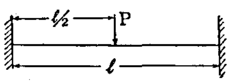


図 43

計算式

$$M_{x=l/2} = \frac{2Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi}{2}\left[\sin\frac{n\pi}{2}-\frac{1}{n\pi}\{1-(-1)^n\}\right]$$

$$EI\theta_{x=l/4} = \frac{2Pl^2}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi}{2}\left\{\cos\frac{n\pi}{4}-\frac{3}{16}+\frac{15}{16}(-1)^n\right\}$$

$$EIY_{x=l/4} = \frac{2Pl^3}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi}{2}\left\{\frac{1}{n\pi}\sin\frac{n\pi}{4}-\frac{9}{64}+\frac{3}{64}(-1)^n\right\}$$

従来の計算値	$M_{x=l/2}=0.125000Pl$	$EI\theta_{x=l/4}=0.015625Pl^2$	$EIY_{x=l/4}=0.002604Pl^3$
級数	$0.113712Pl$	$0.015599Pl^2$	$0.002634Pl^3$
誤差	9.00%	0.16%	1.10%
精度	×	○	○

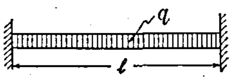


図 44

計算式

$$M_{x=l/4} = \frac{4ql^2}{\pi^3}\left\{\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^3}\sin\frac{n\pi}{4}-\frac{2}{\pi}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^4}\right\}$$

$$EI\theta_{x=l/4} = \frac{4ql^3}{\pi^4}\left\{\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^4}\left(\cos\frac{n\pi}{4}-\frac{1}{2}\right)\right\}$$

$$EIY_{x=l/4} = \frac{4ql^4}{\pi^4}\left\{\frac{1}{n\pi}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^4}\sin\frac{n\pi}{4}+\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^4}\left(-\frac{3}{16}\right)\right\}$$

従来の計算値	$M_{x=l/2}=0.010417ql^2$	$EI\theta_{x=l/4}=0.007813ql^3$	$EIY_{x=l/4}=0.001465ql^4$
級数	$0.010409ql^2$	$0.007818ql^3$	$0.001466ql^4$
誤差	0.07%	0.06%	0.06%
精度	○	○	○

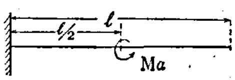


図 45

計算式

$$M_{x=l/4} = \frac{4M_a}{\pi}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n}\sin\frac{n\pi}{4}\cos\frac{n\pi}{8}$$

$$EI\theta_{x=l/4} = -\frac{8M_al}{\pi^2}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^2}\sin\frac{n\pi}{4}\sin\frac{n\pi}{8}$$

$$EIY_{x=l/4} = \frac{16M_al^2}{\pi^3}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^3}\sin\frac{n\pi}{4}\left(\cos\frac{n\pi}{8}-1\right)$$

従来の計算値	$M_{x=l/4}=1.000000M_a$	$EI\theta_{x=l/4}=-0.250000M_al$	$EIY_{x=l/4}=-0.031250M_al^2$
級数	$1.041947M_a$	$-0.249809M_al$	$-0.030996M_al^2$
誤差	4.19%	0.07%	0.80%
精度	△	○	○

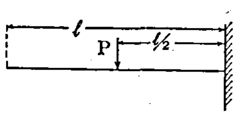


図 46

計算式

$$M_{x=3/4l} = \frac{8Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{3n\pi}{8}\sin\frac{n\pi}{2}\left(\cos\frac{n\pi}{4}-1\right)$$

$$EI\theta_{x=l/4} = \frac{16Pl^2}{\pi^3}\sum\frac{1}{n^3}\cos\frac{n\pi}{8}\sin\frac{n\pi}{2}\left(\cos\frac{n\pi}{4}-1\right)$$

$$EIY_{x=l/4} = \frac{32Pl^3}{\pi^4}\sum\frac{1}{n^4}\sin\frac{n\pi}{2}\left(\cos\frac{n\pi}{4}-1\right)\left(\sin\frac{n\pi}{8}-\sin\frac{n\pi}{2}\right)$$

従来の計算値	$M_{x=3/4l}=$ $-0.250000Pl$	$EI\theta_{x=l/4}=$ $-0.125000Pl^2$	$EIY_{x=l/4}=$ $0.070312Pl^3$
級　数	$-0.249807Pl$	$-0.124666Pl^2$	$0.072839Pl^3$
誤　差	0.07%	0.26%	3.50%
精　度	○	○	○

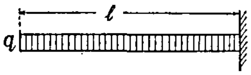


図 47

計算式

$$M_{x=l/4} = \frac{8ql^2}{\pi^2}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^2}\sin\frac{n\pi}{8}\left(\frac{2}{n\pi}-\sin\frac{n\pi}{2}\right)$$

$$EI\theta_{x=l/4} = \frac{16ql^3}{\pi^3}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^3}\cos\frac{n\pi}{8}\left(\frac{2}{n\pi}-\sin\frac{n\pi}{2}\right)$$

$$EIY_{x=l/4} = \frac{32ql^4}{\pi^4}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n^4}\left(\sin\frac{n\pi}{8}-\sin\frac{n\pi}{2}\right)\left(\frac{2}{n\pi}-\sin\frac{n\pi}{2}\right)$$

従来の計算値	$M_{x=l/4}=$ $-0.031250ql^2$	$EI\theta_{x=l/4}=$ $-0.164063ql^3$	$EIY_{x=l/4}=$ $0.083496ql^4$
級　数	$-0.031089ql^2$	$-0.163928ql^3$	$0.083455ql^4$
誤　差	0.52%	0.08%	0.05%
精　度	○	○	○

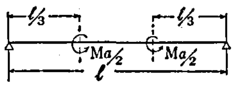


図 48

計算式

$$M_{x=l/4} = -\frac{2M_a}{\pi}\sum\frac{1}{n}\sin\frac{n\pi}{4}\cos\frac{n\pi}{2}\cos\frac{n\pi}{6}$$

$$EI\theta_{x=l/4} = -\frac{2M_al}{\pi^2}\sum\frac{1}{n^2}\cos\frac{n\pi}{4}\cos\frac{n\pi}{2}\cos\frac{n\pi}{6}$$

$$EIY_{x=l/4} = -\frac{2M_al^2}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi}{4}\cos\frac{n\pi}{2}\cos\frac{n\pi}{6}$$

従来の計算値	$M_{x=l/4}=$ $0.250000M_a$	$EI\theta_{x=l/4}=$ $-0.003472M_al$	$EIY_{x=l/4}=$ $0.004340M_al^2$
級　数	$0.238985M_a$	$-0.003200M_al$	$0.004339M_al^2$
誤　差	4.40%	7.80%	0.02%
精　度	△	△	○

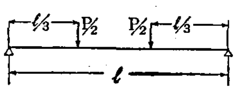


図 49

計算式

$$M_{x=l/4} = \frac{2Pl}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi}{4}\sin\frac{n\pi}{2}\cos\frac{n\pi}{6}$$

$$EI\theta_{x=l/4} = \frac{2Pl^2}{\pi^3}\sum\frac{1}{n^3}\cos\frac{n\pi}{4}\sin\frac{n\pi}{2}\cos\frac{n\pi}{6}$$

$$EIY_{x=l/4} = \frac{2Pl^3}{\pi^4}\sum\frac{1}{n_4}\sin\frac{n\pi}{4}\sin\frac{n\pi}{2}\cos\frac{n\pi}{6}$$

従来の計算値	$M_{x=l/4}=$ $0.125000Pl$	$EI\theta_{x=l/4}=$ $0.039930Pl^2$	$EIY_{x=l/4}=$ $0.012586Pl^3$
級　数	$0.124764Pl$	$0.039942Pl^2$	$0.012587Pl^3$
誤　差	0.18%	0.03%	0.01%
精　度	○	○	○

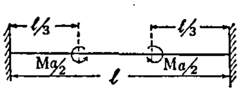


図 50

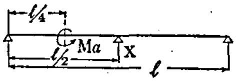
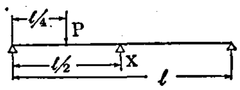
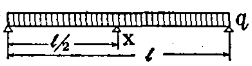
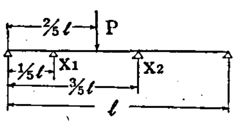
計算式

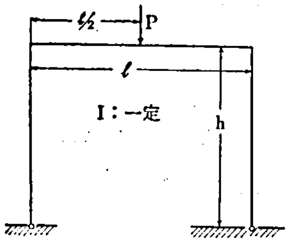
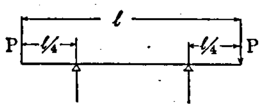
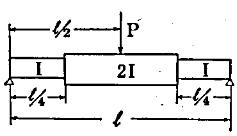
$$M_{x=l/2} = -\frac{M_a}{\pi}\sum_{1\cdot3\cdot5\cdots}\frac{1}{n}\sin\frac{n\pi}{2}\sin\frac{n\pi}{6}\left(\sin\frac{n\pi}{2}-\frac{2}{n\pi}\right)$$

$$EI\theta_{x=l/3} = -\frac{2M_al}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi}{6}\left(\cos\frac{n\pi}{3}-\frac{1}{3}\right)$$

$$EIY_{x=l/2} = -\frac{2M_al^2}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi}{6}\left(\frac{1}{n\pi}\sin\frac{n\pi}{2}-\frac{1}{4}\right)$$

従来の計算値	$M_{x=l/2}=$ $-0.333333M_a$	$EI\theta_{x=l/3}=$ $-0.055556M_al$	$EIY_{x=l/2}=$ $-0.013889M_al^2$

	級　数	$-0.313122M_a$	$-0.051264M_a l$	$-0.014112M_a l^2$
	誤　差	6.00%	7.70%	1.60%
	精　度	△	△	○
 図 51	計算式	$X = \dfrac{\pi M_a \sum \frac{1}{n^3}\cos\frac{n\pi}{4}\sin\frac{n\pi}{2}}{l \sum \frac{1}{n^4}\sin^2\left(\frac{n\pi}{2}\right)}$		
	従来の計算値	$X=2.250000M_a/l$		
	級　数	$2.249871M_a/l$		
	誤　差	0.05%		
	精　度	○		
 図 52	計算式	$X = -\dfrac{\sum \frac{1}{n^4}\sin\frac{n\pi}{4}\sin\frac{n\pi}{2}}{\sum \frac{1}{n^4}\sin^2\left(\frac{n\pi}{2}\right)} P$		
	従来の計算値	$X=-0.687500P$		
	級　数	$-0.687669P$		
	誤　差	0.02%		
	精　度	○		
 53 図	計算式	$X = -\dfrac{ql\sum \frac{\{1-(-1)^n\}}{n^5}\sin\frac{n\pi}{2}}{\pi\sum \frac{1}{n^4}\sin^2\left(\frac{n\pi}{2}\right)}$		
	従来の計算値	$X=-0.625000ql$		
	級　数	$-0.625105ql$		
	誤　差	0.001%		
	精　度	○		
 図 54	計算式	$X_1 = -\dfrac{\begin{vmatrix} k_1 & a_{12} \\ k_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}\cdot P$	$X_2 = -\dfrac{\begin{vmatrix} a_{11} & k_1 \\ a_{21} & k_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}\cdot P$	
	従来の計算値	$X_1=-0.738636P$	$X_2=-0.534091P$	
	級　数	$-0.742715P$	$-0.531661P$	
	誤　差	0.55%	0.45%	
	精　度	○	○	
	計算式	$M = \dfrac{4Pl}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi}{2}\left(\frac{1}{1+\frac{2}{3}\cdot\frac{h}{l}\cdot\frac{I}{I_1}}\right)$		

 図 55	従来の計算値	$M=0.125Pl\left(\frac{1}{1+{}^{2}/_{3}\cdot{}^{h}/_{l}\cdot{}^{I}/_{I_1}}\right)$		
	級　数	$0.125059Pl\left(\frac{1}{1+{}^{2}/_{3}\cdot{}^{h}/_{l}\cdot{}^{I}/_{I_1}}\right)$		
	誤　差	0.04%		
	精　度	○		
 図 56	計算式	$M_{x=l/2}=-\frac{4Pl}{\pi^2}\sum\frac{1}{n^2}\sin^2\left(\frac{n\pi}{2}\right)\cos\frac{n\pi}{4}$ $EI\theta_{x=l/4}=-\frac{4Pl^2}{\pi^3}\sum\frac{1}{n^3}\sin\frac{n\pi}{2}\cos^2\left(\frac{n\pi}{4}\right)$ $EIY_{x=l/2}=-\frac{4Pl^3}{\pi^4}\sum\frac{1}{n^4}\sin^2\left(\frac{n\pi}{2}\right)\cos\frac{n\pi}{4}$		
	従来の計算値	$M_{x=l/2}=-0.25Pl$	$EI\theta_{x=l/4}=-0.062500Pl^2$	$EIY_{x=l/2}=-0.028646Pl^3$
	級　数	$-0.252662Pl$	$-0.062530Pl^2$	$-0.028648Pl^3$
	誤　差	1.06%	0.05%	0.006%
	精　度	○	○	○
 図 57	計算式	$M'=\frac{Pl}{\pi^2}\sum\frac{1}{n^2}\left\{\sin^2\left(\frac{n\pi}{2}\right)+\sin\frac{n\pi}{2}\sin\frac{n\pi}{4}\right\}$ $-\frac{Pl}{4\pi}\sum\frac{2}{n}\sin\frac{n\pi}{2}\cos\frac{n\pi}{4}$		
	従来の計算値	$M'_{x=l/2}=0.125Pl$		
	級　数	$0.121129Pl$		
	誤　差	3.09%		
	精　度	○		

（表中の精度で×印は5項まででは不適当であり，△，○印は適当であることを示す。）

4. む　す　び

本研究は静定・不静定をとわず一般的な荷重状態に適用され，数値計算上で項数を第5項に限定した場合にはどのような支持状態でも，荷重が多くなり複雑になる程精度はよくなり，同じ支持状態では bending moment, slope, deflection の順に精度がよくなるので slope を使う門型ラーメン，deflection を使う連続はりには特に有効である。したがって本理論は広く構造物の解析にも拡張発展ができる。また精度の低い場合には本理論で解析し，級数の収束値表を用いて数値計算もできる。おわりに本研究に貴重な示唆と御指導をいただいた信州大学工学部宮入武夫教授に厚く感謝する次第です。

参　考　文　献

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○ Timoshenko : Strength of Matrials　Part Ⅱ.

○ 関川 : 信州大学工学部紀要　18号